absorbing elements, \( k_{\text{eff}}' \). Here \( k_{\text{eff}}' \) is calculated for a reactor in which the channels formed by withdrawing the absorbing elements are filled with active-zone material. For any value of \( k_{\text{eff}}' < k_{\text{eff}} \), a value was taken for the radius of the absorbing element such that, in the given approximation, the critical equation of the reactor with the system of absorbing elements went to zero. The values of the radii of the absorbing elements were found for the three degrees of approximation \( k = 0, k = 1, \) and \( k = 2 \) (in all cases \( n = 2 \)). To show what effect the number of terms taken in the series in \( n \) has on the radius of the absorbing element, calculations were also made for \( n = 0 \) and \( k = 0 \).

Fig. 2. Change in effectiveness of a system of three absorbing elements as a function of radius of element. Nomenclature same as Fig. 1.

It follows from an examination of the table, and the curves of Fig. 2, that even for large diameters of the absorbing element, the first order approximation \( k = 1 \) gives good accuracy. In the critical equation for the reactor system of three absorbing elements, the angular dependence of the neutron flux in the reactor may be neglected, i.e., we may take \( n = 0 \). Thus, for example, the values of the radius of the absorbing element \( (a_{AE}/R_{\text{core}}) \) at \( k = 1 \) and \( k = 2 \) are equal to 0.1073 and 0.1076 respectively (for \( n = 2 \)), and the curves of Fig. 2 for these approximations run together.

Thus it follows from what we have said, that numerical solutions of the criticality equations can be limited to the two first terms of the series in \( k \) and \( n \) for rather large absorber diameters \( (\sim 0.3 R_{\text{core}}) \).

All the numerous and difficult calculations were made by R. V. Kuleva to whom the author is very much obliged.

<table>
<thead>
<tr>
<th>( k_{\text{eff}}' )</th>
<th>( a_{AE}/R_{\text{core}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k = 0; m = 0, n = 0 )</td>
<td>13.56, 0.1051, 0.1051</td>
</tr>
<tr>
<td>( k = 1; m = 1, n = 2 )</td>
<td>9.56, 0.0737, 0.0737</td>
</tr>
<tr>
<td>( k = 2; m = 2, n = 2 )</td>
<td>9.56, 0.0737, 0.0737</td>
</tr>
</tbody>
</table>

ON THE APPROXIMATE SOLUTION OF THE TRANSPORT EQUATION BY THE METHOD OF MOMENTS

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Our object is to calculate the energy distribution of neutrons at a given distance from a point isotropic source situated in an infinite homogeneous medium. The problem is solved by the method of moments, whose essence consists in the following. From the kinetic equation describing the slowing down and diffusion of neutrons, some of the first even-space moments of the function desired are determined. Afterwards an approximate representation of this function is constructed with the same exact first even moments [1].

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In this paper the representation indicated is constructed by taking into consideration the asymptotic behavior of the solution; the first three moments of the approximating function are used.

Suppose \( \Phi(r) \) is a nonnegative function defined in the interval \((0, \infty)\). We assume that the first three even moments of this function are known:

\[
\mu_n = \frac{1}{n!} \int_0^\infty r^n \Phi(r) \, dr, \quad n = 0, 2, 4.
\]

We take

\[
\Phi(r) = \tilde{\Phi}(r) = Ar^{\nu-1}e^{-ar}
\]

and choose the parameters \( \nu, \alpha \) and \( A \) so that the three first moments of the function \( \tilde{\Phi}(r) \) coincide with the known moments of \( \Phi(r) \). This problem has a solution for every nonnegative function, with all parameters of the representation (1) being real positive numbers defined consecutively from the relations

\[
\nu = \frac{5p - 1 + \sqrt{1 + 14p + p^2}}{2(1 - p)}, \quad p = \frac{\mu_2^A}{6\mu_3\mu_4};
\]

\[
\alpha = \sqrt{\frac{\nu(\nu + 1)\mu_2}{2\mu_4}};
\]

\[
A = \frac{\mu_3\alpha^\nu}{\Gamma(\nu)},
\]

and \( \Gamma(\nu) = \int_0^\infty x^{\nu-1}e^{-x} \, dx \) is Euler's \( \gamma \)-function.

By way of an example of the use of the formula (1), we consider the problem of the space distribution of neutrons emitted by a point isotropic source. For simplicity we assume that the scattering indicatrix is spherically symmetric. The transport equation in this case has the form [2]

\[
\mu \frac{\partial \Phi}{\partial r} + \frac{1 - \mu^2}{r} \frac{\partial \Phi}{\partial \mu} + \Phi = \frac{\omega}{2} \int_{-1}^{+1} \Phi(r, \mu') \, d\mu' + \frac{\delta(r)}{8\pi r^2};
\]