ON PATH-INDEPENDENT INTEGRALS IN THREE-DIMENSIONAL NONLINEAR FRACTURE DYNAMICS*

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ABSTRACT
This paper deals with the path-independent integrals in nonlinear three-dimensional fracture dynamics. Both the nonlinear elastic case and the elastic-plastic case are considered, and some path-independent integrals have been worked out. For explaining the physical meaning of these integrals, a specimen with plane notch is considered, and the relation between the integral and dynamical crack extension force is established. Thus, such integrals may serve as a fracture criterion in nonlinear fracture dynamics.

I. Introduction
In 1968, J.R. Rice [1] proposed the famous $J$-integral in two-dimensional nonlinear elastic fracture statics. Since then, an attractive fracture criterion has been formed on the basis of this integral. We have

$$J = \int_{\Gamma} W \, dy - \int_{\Gamma} \cdot \frac{\partial \mathbf{T}}{\partial \mathbf{x}} \, dS \quad (1.1)$$

where $W$ is the strain energy density, $\Gamma$ is an integration path around the crack tip (Fig. 1), and $\mathbf{T}$ is the traction vector along $\Gamma$. The very important factor of $J$ is its path-independence. Thus, we could consider it as a physical parameter. The proof for path-independence of $J$ is based on the deformation theory of plasticity and this prevents its use to crack propagation problem, in which unloading will take place near the crack tip [2, 3].

Recently, Prof. C. Ouyang [4] proposed some path-independent integrals in nonlinear fracture dynamics, but they are all for the two-dimensional case.

In this paper, we consider the path-independent integrals in three-dimensional nonlinear fracture dynamics. First, we discuss the nonlinear elastic case and propose some such integrals. Then, we consider the elastic-plastic...
case. To explain the physical meaning of these integrals, we consider a plane crack with notched tip, and show that this integral is related to the dynamical crack extension force. Thus, they may serve as a fracture criterion in three-dimensional nonlinear fracture dynamics.

II. Crack Propagation in Nonlinear Elastic Media

Let us consider the nonlinear elastic case first. Suppose there is a plane crack $\gamma$ in a three-dimensional body, as shown in Fig.2. For dynamical problems, we have the equation of continuity and equations of motion as follows:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + X_i = \rho \frac{\partial u_i}{\partial t}$$

(2.1)

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i} \left( \rho \frac{\partial u_i}{\partial t} \right) = 0$$

(2.2)

Here $\sigma_{ij}$ is the stress tensor, $X_i$ is the body force per unit volume, and we suppose it is independent of $x_3$. $\rho$ is the density, $x_i$ is rectangular Cartesian coordinate and $t$ is the time.

Let $S$ be a surface around some part of the crack border (Fig.2): we may establish the following

**Theorem 1.** The vector integral

$$Y = \int_0^t \int_S \left( (W - X u_i - K) \hat{n} - T \hat{\nabla} u_i \right) dS dt + \int_V \rho u_i \hat{\nabla} u_i dV \bigg|_0^t$$

(2.3)

is path-independent for any surface $S$ around some part of the crack border and any $t_1 > t_0 > 0$. Here $\hat{n}$ is the projection of outer normal $\hat{\nu}$ of $S$ on $x, y$ plane. $\hat{\nabla}$ is the operator $i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y}$. The strain energy density

$$W = \int \sigma_{ij} \epsilon_{ij}$$

(2.4)

and $\epsilon_{ij}$, the strain tensor. The kinetic energy density

$$K = \frac{1}{2} \rho v_i \dot{v}_i$$

(2.5)

and $v_i = \frac{\partial u_i}{\partial t}$ is the velocity. The domain $V$ is a region bounded by $S$ and crack surfaces. (Fig.2).

**Proof:**

For the first step, we consider a closed surface $S$, not including the crack border inside. We have

$$\int_S T \hat{\nabla} u_i dS = \int_S \sigma_{ij} \epsilon_{ij} \hat{n} dS = \int_V \frac{\partial}{\partial x_j} \left( \sigma_{ij} \hat{\nabla} u_i \right) dV$$

FIG. 2 Plane Crack in 3-dimensional Body

FIG. 3 The Vector $\hat{\nu}$