THE CONSTRUCTION OF LARGE ELEMENT IN
FINITE ELEMENT METHOD*

Liang Guo-ping (梁国平)
(Institute of Mathematics, Academia
Sinica, Beijing)
Fu Zi-zhi (傅子智)
(Beijing Petroleum Design Institute)
(Received Feb. 27, 1982)

ABSTRACT
In the usual finite element method, the order of the inter-
polation in an element is kept unchanged, and the accuracy
is raised by subdividing the grid denser and denser. Alter-
natively, in the large element method, the grid is kept un-
changed, and the terms of approximate series in the element
are increased to raise the accuracy.
In this paper, a method for constructing large elements
is presented. When using this method, two sets of variables,
one set defined inside the element, and the other defined on
the boundary of the element, are adopted. Then, these two
sets of variables are combined by the hybrid—penalty function
method. This method can be applied to any elliptic equations
in a domain with arbitrary shape and arbitrary complex boun-
dary condition. It is proved with strict mathematical method
in this paper, that in general cases, the accuracy of this me-
thod is much higher than that of the usual element and the
large element method presented in [7]. Therefore, the degrees
of freedom needed in this method are much fewer than those
in the two methods if the same accuracy is preserved.

I. Introduction
The usual element method is to divide the field which is to be solved into
many small domains and then set up interpolation polynomials for each small ele-
ment. If the conditions are satisfied properly, these interpolation polynomials
on small domains will tend to exact solution if the dimensions of all the small
elements tend to zero. Usually, when the finite element method is used to eval-
uate an approximate solution, it is necessary to divide the domain into many
small domains. Since there are many elements and many degrees of freedom, the
amount of the computation and storage requirement of the computer and the input
data is relatively large. So it may take much more time to prepare the data and
to accomplish the calculation using the classical finite element method than using
the classical a...
applied to regular domain and simple equations while the finite element method is fit for arbitrary domain or arbitrary complex equations.

In view of the strong and weak points of the finite element and the analysis methods, the "combined method" which combines these two methods has been developed\(^\text{[1-4]}\). That is to adopt the analysis method on regular domain and to adopt the finite element method on irregular domain. When this method is applied to the problem with singular point and boundless domain, the analysis method is adopted in the neighborhood of the singular point while the finite element method is adopted in another domain. As the analysis method can calculate much more accurately in the neighborhood of the singular point than the finite element method\(^\text{[4]}\), it is very effective and is used earlier for this kind of problems\(^\text{[5]}\).

A great difference between the finite element and the analysis method is that, with the finite element method the accuracy is raised by subdividing the grid denser and denser, while with the analysis method the accuracy is raised by adding more and more terms of the series expansion successively. So the latter method is more convenient. Ref.\(^\text{[6]}\) presents a method of adopting series expansion even on the irregular domain, that is, the so-called large element method. This method comprises the classical analysis which is somewhat different from the analysis and the large element method is different from the finite element method. It tends to exact solution by increasing successively the terms of the series expansion in the element. The large element can be constructed on irregular domain and the equation need not be satisfied. Ref.\(^\text{[7]}\) presents a finite element method completely constructed with large elements. In spite of the fact that this kind of method has recently been used in the field of engineering, ref.\(^\text{[7]}\) is the first one which gives the theoretical analysis of the method. Triangular elements and polynomial expansion are adopted in that paper. If the compatible conditions are satisfied for each element, the error estimate is obtained. But the main shortcomings of that reference are: 1). Only polynomial series are used. As the smoothness of the exact solution near the corner (especially near the concave corner) is rather poor and the approximate accuracy is insufficient, the amount of computation and the storage requirement may not be less than that of the finite element method theoretically. 2). The application of the method is strictly limited for adopting compatible elements. If different series expansions are defined on each of two neighbouring domains, it is very difficult to satisfy the interelement compatible condition which has to be satisfied. Then, the first shortcoming is inevitable, i.e., polynomial series of the same order have to be adopted. And secondly, it is very difficult to apply this method to equations of high order (e.g., questions of plates and shells), domains with curvilinear boundaries and other complex problems.

This paper presents a new method for constructing a large element in the fi-