J. Aczél and J. Dhombres, Functional Equations in Several Variables

Functional equations are, roughly speaking, equations in which the unknown (or unknowns) are functions. The words "several variables" in the title mean that in this book only such equations are dealt with in which the number of variables is greater than the (total) number of places in the unknown function(s). This excludes iterative equations, differential, difference-differential, difference and similar equations containing in most cases just one variable if the unknown function is of one variable.

Although functional equations can already been found in the works of Euler, D'Alembert, Poisson, Cauchy, Abel, Weierstraus, Darboux and Hilbert the theory of functional equations developed only in this century. This development accelerated in the fifties and an important milestone in this process was the appearance of the monograph of J. Aczél, "Vorlesungen über Funktionalgleichungen und ihre Anwendungen", Birkhäuser Verlag 1961; extended English translation "Lectures on functional equations and their applications", Academic Press 1966) in which a systematic presentation of the field was given and the foundation of a modern theory of functional equations was laid down. The present book can, to a certain extent, be considered as the continuation of the above mentioned work. It deals with the theory of functional equations in several variables with the main emphasis on the applications in and outside of mathematics. Such areas of applications are, among others: combinatorics, probability and information theory, economics, decision making and mean values, almost periodic functions and harmonic analysis, operator theory, geometry, physics, groups, grupoids and semigroups, number theory. The required prerequisites are kept to a minimum, only calculus, general and linear algebra and basic Lebesgue theory of integration is supposed.

The book contains 21 chapters some of which can be read independently from each other. To indicate some further results and directions of research the authors include more than 400 items as 'excercises and further results' at the end of the chapters. For some selected exercises and results hints are given.

The first Chapter deals with the axiomatic foundation of vector addition using d'Alembert's functional equation.

In Chapter 2 and 3 the theory of the most important functional equations, the basic Cauchy's equations, is given.

The theory of Cauchy's equations generalized to several multiplace vector and matrix functions and complex functions is developed in Chapters 4 and 5. This theory is applied to
the characterization of differential geometric objects, to harmonic analysis and to recursive entropies.

Chapters 6 and 7 are devoted to conditional Cauchy equations and extension theory. Applications to characterizations of collineations on the real plane and to concensus allocations are treated.

Chapter 8 deals with d'Alembert's functional equation and its application to non-euclidean mechanics.

Chapters 9, 10, 11 and 12 furnish further applications in functional analysis (characterization of strictly convex spaces, characterizations of inner product spaces), gas dynamics, combinatorics and Markov chains.

In Chapter 13 and 14 the theory of the well-known functional equations of the trigonometric functions and their generalizations are treated and it is shown how regularity conditions can subsequently be relaxed.

The main result of Chapter 15 is a uniqueness theorem and its applications to quasiarithmetic means.

Chapter 16 gives applications to additive number theoretical functions and to coding theory.

Chapters 17, 18 and 19 are devoted to special composite type functional equations like mediality, self-distributivity, with applications to mean values, webs, nomograms.

In chapter 20 the generalized homogeneity equation and its application to economics is dealt with.

Chapter 21 contains historical notes and comments with an overlook of present aspirations and applications of this theory.

An extensive bibliography containing more than 1500 items completes the monograph.

This book is well-organized and was written with great pedagogical skill which makes the content understandable even for the beginners. It can be warmly recommended for those (researches or beginners) who are interested in the theory or applications of functional equations.

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The subtitle of the book is "A Collection of Essays", which is only "half-true", because half of the book is mathematics while the remaining half are indeed "essays". The mathematical part of the book is written by three outstanding geometers. The first paper entitled "On the geometric flat embedding of abstract complexes with symmetries" (49 pages) is due to J. Bokowski. After two introductory sections, the author studies symmetries in computational synthetic geometry (the titles of subsections: Dyck's regular map; chirotopes with symmetries; symmetrical embeddings of Möbius' torus; completely hidden symmetries for convex polyhedra; symmetric chirotope-manifolds with minimal number of vertices; a convex polyhedron with hidden symmetries). The paper ends with a comprehensive list of references. The paper seems to be a good supplement to the book of J. Bokowski and B. Sturmfels, "Computational synthetic geometry", Springer Lect. Notes in Math., 1989.