BIBASIC ANALYTIC FUNCTIONS AND DISCRETE 'BIBASIC' HYPERGEOMETRIC SERIES

M. A. KHAN (Aligarh)

Abstract

This paper deals with a study of a class of functions called 'bibasic analytic functions'. Using discrete power \( z^{(n)^m} \), discrete bibasic hypergeometric functions have been introduced.

§ 1. Introduction

In 1972, a theory of discrete analytic functions was developed by Harman [6] on the geometric lattice

\[
H = \{(q^mx_0, q^ny_0); m, n \in \mathbb{Z}, \; 0 < q < 1; (x_0, y_0) \text{ fixed in } \mathbb{C}\}
\]

in the complex plane.

Two operators \( R_q \) and \( R_p \) are defined with,

\[
R_q f(z) = \bar{z} f(z) - x f(x, qy) + iy f(qx, y)
\]

and

\[
R_p f(z) = \bar{z} f(z) - x f(x, py) + iy f(px, y)
\]

where \( p = q^{-1} \) and \( f : H \rightarrow \mathbb{C} \).

\( R_q f(z) \) and \( R_p f(z) \) are respectively called the \( q \)- and \( p \)-residues of the function at \( z \). If the \( q \)-residue (\( p \)-residue) of \( f \) is zero at \( z \); \( f \) is said to be \( q \)-analytic (\( p \)-analytic) at \( z \).

In establishing his theory Harman used discrete derivatives of \( f \) at \( z \). Thus

\[
D_{q,x} f(z) = \frac{f(x, y) - f(qx, y)}{(1 - q)x} \quad \text{and} \quad D_{q,y} f(z) = \frac{f(x, y) - f(x, qy)}{(1 - q)iy}
\]


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where $f$ is discrete function.

When $D_{q,x} f(z) = D_{q,y} f(z)$, we write $D_q f(z)$ for both and $f$ becomes $q$-analytic at $z$.

Similarly

$$D_{p,x} f(z) = \frac{f(x, y) - f(px, y)}{(1 - p)x} \quad \text{and} \quad D_{p,y} f(z) = \frac{f(x, y) - f(x, py)}{(1 - p)y}$$

(1.5)

where $p = q^{-1} > 1$.

When $D_{p,x} f(z) = D_{p,y} f(z)$, we write $D_p f(z)$ for both and then $f$ is $p$-analytic at $z$.


In 1982, Velukutty [13] investigated functions which are both $q$- and $p$-analytic in certain domain in the discrete geometric space. He named the solution as bianalytic functions. Thus bianalytic functions satisfy the equation $R_q f(z) = R_p f(z) = 0$ everywhere in certain domain.

The present paper deals with a study of a class of functions which we call 'bibasic analytic' functions. These functions are analogous to analytic functions and as such the resulting theory of 'bibasic analytic' functions is an extension of the theory of $q$-analytic functions or $p$-analytic functions due to Harman [6] and bianalytic function due to Velukutty [13].

A suitable lattice, for such functions has been constructed which involves two unconnected bases $p$ and $q$ to define complex valued functions $f(z)$ only over its points. It has also been demonstrated how successfully most of the results and concepts given by Harman [7, 8] for $q$-analytic functions can be extended in the case of 'bibasic analytic' functions. The discrete power evolved for 'bibasic analytic' functions, helps in defining the discrete hypergeometric functions with two unconnected bases. We call such a function 'discrete bibasic' hypergeometric functions.

§ 2. The lattice

Let the complex number $z$ be designated by its components $(x, y)$, e.g.

$$z \equiv (x, y), \quad z_0 \equiv (x_0, y_0), \quad z_i \equiv (x_i, y_i).$$

(2.1)

The set of lattice points,

$$Q' = \{(p^m x', q^n y'), m, n \in I, \text{ the set of integers}\}$$

(2.2)

represents the 'discrete plane' $Q'$ with respect to some fixed point $z' = (x', y')$ in the first quadrant. Extensions to other quadrants are easy.

Two lattice points $z_i, z_{i+1} \in Q'$ are said to be 'adjacent' if $z_{i+1}$ is one of the points $(px_i, y_i), (p^{-1}x_i, y_i), (x_i, qy_i)$ or $(x_i, q^{-1}y_i)$. 