Condensing Generalized Polynomials

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Abstract. An extension of geometric programming to include generalized polynomials, and not only positive polynomials, is described, together with an algorithm and a numerical example.

1. Introduction

Considerable interest in optimization has developed over the past decade. As a result, many new optimization techniques have been found and applied to problems arising in managerial decision making. However, not much has been done in the field of optimizing engineering design problems. In the time-honored approach to optimizing engineering design, a series of parametric curves were plotted and, by cross comparison, the optimum design was found. In recent years, such problems were solved by digital computers using mainly exhaustive search. The reason for such a gap between the theory and the application of optimization techniques lies perhaps in the inherently nonlinear relations which exists between the design parameters and the operating specifications.

In the past several years, this gap was narrowed with the introduction of geometric programming, found by Duffin, Peterson, and Zener (Ref. 1). Problems which can be formulated as sums of positive terms, called positive polynomials or posynomials, can readily be solved by this new method. It was shown that such problems can be transformed into convex programming problems and can then be solved by techniques applicable to convex programming, such as the cutting plane (Ref. 2), gradient projection (Ref. 3), or separable programming (Ref. 4).

The more general case where the objective and constraint functions

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are given by generalized polynomials (Ref. 5) cannot be handled by geometric programming.

A class of techniques for transforming such problems to a sequence of geometric programs is described, and it is shown that, in a special case, the algorithm reduces to the one developed recently by Avriel and Williams (Ref. 6). The results obtained enable the faster use of the above algorithm since, in the expansion of posynomials to a single-term posynomial, the evaluation of derivatives is not required.

The main idea underlying this class of techniques is described by Passy (Ref. 7). The idea used for this particular problem was called condensation by Duffin (Ref. 8). Therefore, the term condensation is used in throughout this work. However, a better term is expansion, as illustrated in Ref. 7.

The solution obtained by this method is a pseudominimum point (Ref. 5), which in many cases turns to be a local minimum (Ref. 6). For further details, see Section 7.

2. Geometric Programming and Generalized Polynomial Optimization

Generalized polynomial optimization is a nonlinear programming technique for solving the following problem:

\[ \min P_0(x), \quad P_0(x) > 0, \tag{1} \]

where \( x \) is constrained by

\[ x \in (\mathbb{R}^m)^+, \tag{2} \]

\[ P_k(x) \leq a_k, \quad k = 1, \ldots, P. \tag{3} \]

The functions \( P_k(x) \) are generalized polynomials, that is,

\[ P_k(x) = \sum_{i=m(k)}^{n(k)} D_k \prod_{j=1}^{m} x_i^{\beta_{ij}}, \tag{4} \]

where \( m(0) = 1, \quad m(1) = n(0) + 1, \ldots, \]

\[ m(k) = n(k - 1) + 1, \ldots, \quad m(P) = n(P - 1) + 1. \tag{5} \]

The coefficients \( D_k, \beta_{ij} \) are given real numbers, and \( a_k \neq 0 \). This class of problems is a very general one, since almost every practical design problem is approximated by generalized polynomials.