BEHAVIOR OF SMALL PLASMA BUNCHES IN A WAVEGUIDE AND INTERACTION WITH CONDUCTING WALLS

G. A. Askarian

The examination of conditions which provide isolation and repulsion of bunches of a quasi-neutral plasma from the walls of a waveguide are of interest in connection with a proposal made by Veksler [1] concerning methods of radiation acceleration of a bunched plasma; these methods are proposed for enhancing the radiation and for obtaining Coulomb entrainment of the ions by electrons. Such isolation conditions are also important in a consideration of various methods of injection and compression of plasma and in the transport of plasma bunches through a tubular conductor when the bunches carry magnetic fields. An investigation of the radial stability of small bunches close to the axis of a waveguide has been carried out by M. L. Levin and the author [2]. In [2] consideration was also given to the axial stability and confinement of plasma bunches propagating along the axis of distributed magnetic fields.

In the present paper we compute the radial force which acts on a small plasma bunch in a field made up of simple waves when the bunch is displaced by a distance which exceeds the dimensions of the bunch; we estimate the interaction forces between the bunch and conducting walls and examine certain possible configurations of tubular conductors and reflectors.

In analyzing the behavior of a bunch in a waveguide one must remember that when the induced dipole moments of the bunch are small (for example, when the bunch is small or is characterized by a small dynamic polarizability) the interaction of the bunch with the walls is small compared with the interaction with the wave, even close to the wall; these considerations follow from the fact that the effect of the wave is proportional to the first power while the interaction with the wall is proportional to the square of the small dipole moment. In considering such small bunches it is feasible to compute the radial forces which act on the bunch for any displacement from the axis.

The expressions for axially symmetric fields (H_0 and E_0 waves) can be written in the following common notation (the left-hand terms in the curly brackets refer to the H_0-wave):

\[
\begin{align*}
\{H_z; E_z\} &= \{H_a; E_a\} J_0(xr) \sin(\omega t - h z), \\
\{H_r; E_r\} &= \{H_a; E_a\} \frac{k}{k} J_1(xr) \cos(\omega t - h z), \\
\{E_r; H_r\} &= \{-H_a; E_a\} \frac{k}{k} J_1(xr) \cos(\omega t - h z),
\end{align*}
\]

where \(H_a\) and \(E_a\) are the wave amplitudes along the axis of the waveguide, \(x\) is the distance from the axis, \(\chi\) and \(h\) are the transverse and longitudinal wave numbers (\(\chi^2 + k^2 = k^2\)) which are related to the radius of the waveguide \(R\) by the boundary conditions: \(J_\chi(xR) = 0\), \(\chi = \frac{2.8}{R}\) for the H_0-wave and \(J_\chi(xR) = 0\), \(\chi = \frac{2.4}{R}\) for the E_0-wave.

In the H_0 case, the time averages of the radial forces which act on the electric (P) and magnetic (M) dipole moments of the bunch are of the form:

\[
\begin{align*}
(F_P)_{\text{av}} &= \left( \frac{\partial E_r}{\partial \rho} \right)_{\text{av}} = a^2 \frac{h_a^2}{a} \frac{k^2}{k} J_1 J_1 \cos \varphi_P, \\
(F_M)_{\text{av}} &= \left( M_r \frac{\partial H_r}{\partial \rho} + M_z \frac{\partial H_z}{\partial \rho} \right)_{\text{av}} = \\
&= \frac{3}{2} \frac{h_a^2}{a} \left\{ \frac{k^2}{k} J_1 J_1 + J_a J_a \right\} \cos \varphi_M.
\end{align*}
\]
In these expressions $c$ and $\beta$ are the dynamic electric and magnetic polarizability of an isotropic bunch while $\psi_M$ and $\psi_p$ are the phase shifts between the electric and magnetic moments of the bunch and the field (in particular, for a shielded quasi-spherical bunch of radius $a$, $\alpha = - 2\beta = a^3$ and $\psi_p = \psi_M = 0$); $J$ and $J'$ are the Bessel function of argument $\alpha r$ and its derivative with respect to $r$ (below we shall make use of the well-known relations: $J_0' = - x J_0$, and $J_1' = - \frac{1}{r} J_1 + x J_0$).

The total radial force which acts on the bunch is

$$F_r = - \frac{1}{2} \frac{H^2 a}{a} \left\{ \frac{\alpha}{\alpha r^2} J J_1' \cos \psi_p + \beta \left( \frac{\alpha}{\alpha r^2} J J_1' + J J_0' \right) \cos \psi_M \right\}.$$

Close to the axis $J_0 J_0' = - \frac{1}{2} x^2 r$, and $J_1 J_1' = \frac{1}{4} x^2 r$ so that the condition for axial stability is of the form:

$$a k^2 \cos \psi_p + \beta (k^2 - 3 x^2) \cos \psi_M < 0.$$

Close to the wall $J_0 J_0' = x^2 \frac{J_0}{\alpha} \rho$ and $J_1 J_1' = - x^2 \frac{J_1}{\alpha} \rho$ ($\rho = R - r$) hence the condition that the bunch remain far from the wall is

$$a k^2 \cos \psi_p + \beta (k^2 - 2 x^2) \cos \psi_M > 0.$$

For example, whereas a dense plasma bunch can be unstable with respect to the axis of a H$_o$ wave, the field of this wave close to the wall of the waveguide can isolate the bunch from the wall, effectively supplying a force

$$F_r = - \frac{a^3}{32} H^2 a \left( \frac{k^2}{a^2} + x^2 \right) \rho.$$  An analysis of the nature of the force distribution shows that inside the waveguide there is an equilibrium radius at which the position of the bunch is stable. In this case the bunch is contracted by the wave magnetic field, the direction of which oscillates or rotates as a consequence of the phase shift between the $H_1$ and $H_2$ components.

It is also possible to have various combinations of wave fields and fixed external magnetic fields which are distributed over the path of the bunch in such a way that in its coordinate system the magnetic field changes rapidly, rotating or changing direction, and thus providing confinement and stability of a moving plasma bunch.

We now estimate the interaction force between a bunch and a conducting wall, assuming that the distance between the bunch and the wall is greater than the dimensions of the bunch but much smaller than the radius of the waveguide. If the conductivity of the wall is high, the field due to the surface charges and currents, which arise when the polarized bunch is in the proximity of the wall, can be replaced by the image field. The longitudinal and transverse components of the magnetic image of an alternating or moving magnetic dipole are related to the components of the magnetic moment itself by the relations: $M_\parallel = M_\parallel$ and $M_{\perp} = - M_{\perp}$ for the electric dipole image we have $P_{\parallel} = - P$ and $P_{\perp} = P$. The force components normal to the surface of the wall are

$$F_n = \frac{3}{16 \rho^2} \left\{ 2 (M_{\perp} - P_{\perp}) + (M_{\parallel} - P_{\parallel}) \right\} \text{av}.$$

In the $H_0$ case only the longitudinal component $H_z$ is nonvanishing; this acts on the longitudinal component of the magnetic moment so that $F_n = \frac{3}{32 \rho^2} \beta^2 H_0^2 a^3$. This repulsive force increases sharply as the bunch approaches the wall. A comparison of the interaction forces between a dense bunch and a wave and with the wall indicates that when

$$\rho > \frac{a}{\sqrt{a^2 (k^2/2 + x^2)}}$$

the separating force of the field is greater than the repulsive force due to the wall.

We now consider the behavior of a small bunch in a $E_\theta$ field. The average radial forces acting on the bunch are