We denote by $C_0(A)$ the subspace in $C(A)$ consisting of those $x \in C(A)$ for which $x(t_0) = 0$. It is easy to see that the space $C_0(A)$ is complemented in $C(A)$ and isomorphic with $(l^\infty \times l^\infty x \times \cdots)_{c_0}$. Consequently the space $(l^\infty \times l^\infty x \times \cdots)_{c_0}$ is isomorphic with a complemented subspace in $C(T)$.

**Corollary 1.** Let $S$ and $T$ be metrizable spaces. If $S$ is locally compact, countable at infinity, and $T$ is not locally compact, then the spaces $C(S)$ and $C(T)$ are not isomorphic.

**Corollary 2.** The spaces of bounded continuous functions on the real line and on the set of irrational numbers $C(\mathbb{R})$ and $C(\mathbb{Q})$ are not isomorphic.

**Corollary 3.** The space $(l^\infty \times l^\infty x \times \cdots)_{c_0}$ is not isomorphic with the space $(c_0 \times c_0 x \times \cdots)_{c_0}$.

**LITERATURE CITED**


**EXTENSION THEOREMS WITH PRESERVATION OF LOCAL APPROXIMATION**

**PROPERTIES OF FUNCTIONS IN THE NONISOTROPIC CASE**

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Theorems are proved on the extension of functions from a set of sufficiently general form with preservation of the order of local approximation characteristics of these functions, called the $(a,p)$-modulus of continuity. Extension is realized by a linear operator. As corollaries, descriptions are obtained of traces of functions from spaces $B_p^{\sigma}$ and BMO on compacta of sufficiently general form.

1. The goal of the paper is an account of results on extension of a function from a set of sufficiently general form, preserving the order of the local approximation characteristic of this function, called the $(a,p)$-continuity modulus. Since in terms of this characteristic there is defined a whole series of important spaces, in particular, the nonisotropic spaces $B_p^{\sigma}$ and BMO (cf. [1] in connection with the isotropic situation), as corollaries of the basic theorems we get the characteristic of traces of functions from the spaces indicated on compacta of sufficiently general form.

2. Let $\ell = (\ell_1, \ldots, \ell_n)$ be a vector with positive coordinates such that $\sum_{i=1}^{n} \ell_i = n$. We set $g_\ell(x) = \max_i |x_i|/\ell_i$, and by the $\ell$-cube with center $x$ and diameter $\sigma$ we mean the set $Q = \{ y \in \mathbb{R}^n : g_\ell(y-x) < \sigma \}$.

Obviously, $|Q| = \sigma^n$; we say that the measurable set $F, F \subset \mathbb{R}^n$, is $\ell$-regular if

$$\lambda_\ell(F) = \inf_Q \frac{|Q \cap F|}{|Q|} > 0,$$

where the \( \inf \) is taken over all \( l \)-cubes with centers in \( F \) and diameters \( \leq \text{diam} F \). Further we define the local best approximation \( E_{\alpha}(f; A) \in L_q(F) \) and \( A \subset R^n \)

\[
E_{\alpha}(f; A)_{L_q(F)} = \inf_{\rho} \| f - \rho \|_{L_q(A \cap F)},
\]

where \( \rho \) runs through the set of polynomials of degree \( \alpha_i - 1 \) in the variable \( x_i, \ i = 1, \ldots, n \). We set further

\[
E_{\alpha}(f; A)_{L_q(F)} = | A \cap F |^{-1/q} E_{\alpha}(f; A)_{L_q(F)} .
\]

**Definition 1 ([1]).** By the \((\alpha, \rho)\) -continuity modulus of the function \( f \in L_q(F) \) is meant the function \( \omega_{\alpha, \rho}(f, \cdot)_{L_q(F)} \) defined by the formula

\[
\omega_{\alpha, \rho}(f, \tau)_{L_q(F)} = \sup_{\tau} \{ \sum_{\in \pi} | \tau \cap F |^{-1/p} E_{\alpha}(f, \tau)_{L_q(F)} \}^{1/p},
\]

where \( \pi \) runs through families composed on congruent pairwise disjoint \( l \)-cubes of diameter \( \leq \tau \) and with centers in \( F \). Here \( 0 < \rho \leq \infty \), while for \( \rho = \infty \) the right side of (3) is replaced by \( \sup_{\tau} E_{\alpha}(f, \tau)_{L_q(F)} \), where the sup is taken over all \( l \)-cubes of diameter \( \leq \tau \) and with centers in \( F \).

The connection between the quantity (3) and the usual moduli of continuity is established by the result below, carrying over to the nonisotropic situation the corresponding theorem of Brudnyi (cf. [1] and also [2]). In the theorem formulated we set for \( f \in L_p(Y) \)

\[
\omega_{\alpha}^{\ell}(f, \tau)_{L_p(Y)} = \sum_{i=1}^n \omega_i \in \Omega_i (f, \tau^{1/\alpha_i})_{L_p(Y)},
\]

where the \( i \)-th summand on the right is the modulus of continuity of order \( \alpha_i \) in the direction of the axis \( x_i \) (cf. [3, p. 251]).

**Theorem 1.** If \( Y \) is a domain satisfying the strong \( l \)-horn condition (cf. [3, p. 117]) and if \( f \in L_p(Y) \), \( 1 \leq p \leq \infty \), then for \( \alpha \geq \ell \)

\[
\omega_{\alpha, \rho}(f, \tau)_{L_p(Y)} \leq \omega_{\alpha}^{\ell}(f, \tau)_{L_p(Y)}, \quad 0 \leq \tau \leq \text{v}.
\]

The constants do not depend on \( f \) and \( \tau \).

3. We proceed to the formulation of the extension theorem. Let us assume that \( F \) is an \( l \)-regular compactum, and the function \( f \) from \( L_q(F) \) \( (0 \leq q \leq \infty) \) is such that

\[
\omega_{\alpha, \rho}(f, \tau)_{L_q(F)} < \varphi(\tau), \quad 0 < \tau \leq \text{diam} F,
\]

where \( \varphi \) is a monotone increasing function, \( \varphi(0) = 0 \). For the given \( \varphi \) and \( \rho \) we construct the function

\[
\tilde{\varphi}(\tau) = \tau \left\{ \int_0^\tau \frac{\varphi(s)}{s^{1/p}} \frac{ds}{\delta} \right\}^{1/p},
\]

where \( \delta = \sum_{i=1}^n a_i / b_i \), \( 0 < \rho \leq \infty \) (replacing the curly brackets by \( \max_{\tau \in [0, \infty)} \frac{\varphi(\tau)}{\delta^{1/p}} \) for \( \rho = \infty \)); here we assume that \( \varphi(\tau) = \varphi(\text{diam} F) \) for \( \tau > \text{diam} F \).