On Randomness and Thermodynamics

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The past decade has witnessed significant advances in our understanding of nonlinear systems, both theoretically and experimentally. In turn, these insights have led to applications over a broad range of subjects, as well as to new interpretations of complex behavior. Inevitably, it seems, there has been a concomitant rush to re-interpret various well-understood aspects of thermodynamic behavior in many-body systems by means of these mechanisms. It is perhaps useful, therefore, to examine these notions critically, and in doing so we are able to illuminate somewhat the origins of, and motivations for, various dubious descriptions of these systems. Among other things, one notices in such discussions an unfortunate focus on "randomness," rather than on the actual role of probabilities in describing the phenomena. Although valuable insights into some areas of complex behavior have arisen from studies of so-called "deterministic chaos," we can detect no influence they may have on the predictions of statistical mechanics, and thus conclude that they are of little relevance to the thermodynamic behavior of large systems.

1. INTRODUCTION

Attentive students of history have often noted the cycle

Innovation ↔ Discovery ↔ Knowledge

at the heart of human progress. Each time a technological breakthrough occurs there inevitably follows a period of discovery stimulated by these new tools, and then an interval during which a substantial amount of knowledge is accumulated and consolidated prior to another round. It is rather apparent that during the past two decades an extraordinary example of this scenario has evolved with respect to the computer and its application to studies of nonlinear dynamical systems, an area of investigation

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which had been somewhat stymied since the turn of the century. From the early work of Poincaré and Birkhoff, through more recent works of Smale, Lanford, Arnol’d, and others, the deep mathematical difficulties had yielded only grudgingly. But with the enormous technological advances in electronic computing it was possible to focus primarily on the systems rather than on the analytical problem. Although the nonlinear mathematics remains formidable, the new insights into the physics provided by high-speed processors, large-scale memories, and high-resolution graphics are plentiful.

It had been known for a long time that there might exist a number of interesting solutions to nonlinear equations of motion, of course, but the newfound ability to study these in detail has revealed just how strange some can be. Of particular interest are those solutions exhibiting an extreme sensitivity to initial conditions, leading to the notion of “deterministic chaos.” Our fundamental belief that we can accurately predict solutions of deterministic equations of motion by specifying the initial conditions has become much less firm in general, and leads to considerable practical difficulties. Among the latter, the vagaries of weather prediction have been recognized for a long time. One must now be aware of the existence and importance of strange attractors under certain circumstances, as well as Liapunov stability conditions, etc. It is thus evident that conditions may exist in many deterministic systems which allow only a probabilistic estimate of trajectories.

These new insights into nonlinear behavior provide us with an appreciation of just how little is needed to produce serious complexity in many common physical systems. In turn, these advances have led to experiments which emphasize the presence of nonlinear phenomena, and one is now aware of the possibility of “chaotic resonances” in many physical systems (e.g., helium rolls and Couette–Taylor flow). As always in such cycles, one next hopes to systematize detailed studies of very complicated structures guided by these ideas, and thus acquire an understanding of their organizing principles. Many areas of research, in which detailed understanding had been deemed hopeless not too long ago, now see the distinct possibility of realistic model building. Physiology and medicine, and various of the “soft” sciences, come immediately to mind.

2. WHAT OF STATISTICAL MECHANICS?

There are those for whom the complicated behavior seen in otherwise-simple dynamical systems is reminiscent of many-body systems and the necessary statistical descriptions associated with them. Indeed, some have