APPLICATION OF NOMOGRAMS OF EQUIVALENT POINTS IN THE KINEMATICS OF NUCLEAR REACTIONS

G. N. Potetyunko

The representation of kinematic relations in nuclear reactions at nonrelativistic energies by nomograms of equivalent points is considered. Working nomograms enabling all kinematic problems in reactions involving the yield of two particles and some extremely important problems (transformation of cross sections and continuous spectra from the laboratory system of coordinates into the center-of-mass system) for reactions involving the yield of three or more particles to be solved are presented. The nomograms not only mechanize a great deal of the calculations, but also provide a picture of the physical aspects of scattering following from kinematical considerations and show how measuring errors affect the error in determining the remaining kinematical parameters.

The nomograms under consideration are extremely convenient for practical use, since they are quite simple and give high accuracy; a particularly important point is that they also give a clear geometric representation of all the special features in the functional relationships being studied.

The authors of [1, 2] published a nomogram for the system of equations [3]

\[ \begin{align*}
\cot \theta_1 &= \frac{p_1 \cos \phi}{\sin \theta} ; \\
\cot \theta_2 &= \frac{p_2 \cos \phi}{\sin \theta}
\end{align*} \]

describing the interrelation between the escape angles of the products of nuclear reaction I + II - 1 + 2* in the laboratory or L system (\(\delta_1\) and \(\delta_2\)), the escape angle in the center-of-mass or C system (\(\delta\)), and the beam energy \(E_I\). The quantities \(p_{1(2)}\) constitute the ratio of the translational velocity to the velocities of particles 1 and 2 in the C system and are calculated from the formulas [3]

\[ \begin{align*}
p_{1(2)} &= \sqrt{\frac{A_{1(2)}E_I}{E_I + QB}} ; \\
A_{1(2)} &= \frac{m_{1(2)}m_{11}}{m_{1}^2} ; \\
B &= 1 + \frac{m_1}{m_{11}} ; \\
p_1 &= \frac{p_1}{m_1}.
\end{align*} \]

A weak point of the nomogram given in [2] is the fact that values of \(\delta_{1(2)}\) close to 0 and \(\pi\) do not appear on the \(\delta_{1(2)}\) scale. Figure 1 (a and b) presents nomograms obtained from those mentioned by a projective transformation effected by means of formula (6) (see [1]) with \(\lambda = 1/\sqrt{24}\) (Fig. 1a) and \(\lambda = 1/\sqrt{25}\) (Fig. 1b). Values of \(\delta_{1(2)}\) close to \(\pi/2\) do not appear on these nomograms. Thus the nomograms of [2] supplement those derived in the present paper. The advantage of these is not only that they mechanize the computing process but also that they reflect some of the aspects of the reaction in a clear geometric form. This aspect is fully treated in [4], and we shall therefore here concentrate mainly on aspects omitted from the earlier paper.

Let us first consider the critical points of the nomogram, the intersections of its scales. In our case the scales intersect at \(p_1 = 1\) and \(p_2 = 1\). These values of \(p_1\) and \(p_2\) correspond to the following values of beam energy, which we shall call critical:

\[ E_{1(2)}^{(1,2)} = \frac{m_{1(2)}Q}{m_{1(2)}^2 - m_{1(2)}}. \]

* Subsequently index 1 means the light reaction product and index 2 the heavy reaction product. On this condition \(p_1 \leq p_2\).

The values $\rho_1 = 1$ and $\rho_2 = 1$ separate single- and double-valued regions in the relation between angles $\delta_1$ and $\delta_2$. For $\rho_1 < 1$ and $\rho_2 < 1$, the relationship between the angles $\delta_1$ and $\delta_2$ is mutually single-valued or unambiguous. For $\rho_1 < 1$ and $\rho_2 > 1$ to one escape angle $\delta_1$ of the light reaction product there corresponds one escape angle $\delta_2$ of the heavy reaction product, but to one angle $\delta_2$ there correspond two angles $\delta_1$. For $\rho_1 > 1$ and $\rho_2 > 1$ the relationship between $\delta_1$ and $\delta_2$ is mutually double-valued. The relations between $\rho_1$ and $\rho_2$ for various conditions are shown in Tables 1 and 2.