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Computability and Physical Theories

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The familiar theories of physics have the feature that the application of the theory to make predictions in specific circumstances can be done by means of an algorithm. We propose a more precise formulation of this feature—one based on the issue of whether or not the physically measurable numbers predicted by the theory are computable in the mathematical sense. Applying this formulation to one approach to a quantum theory of gravity, there are found indications that there may exist no such algorithms in this case. Finally, we discuss the issue of whether the existence of an algorithm to implement a theory should be adopted as a criterion for acceptable physical theories.

"Can it then be that there is... something of use for unraveling the universe to be learned from the philosophy of computer design?"

—J. A. Wheeler[1]

1. INTRODUCTION

The process of making predictions in physics involves two steps. The first is the formulation of a theory covering a broad class of phenomena; the second, the application of that theory to the specific circumstances of interest. For making predictions of the motion of the planets of our solar system, for example, the first step is the formulation of Newton's theories of mechanics and gravitation; the second the solution of a system of many dozens of ordinary differential equations. These two steps are rather different in character. The first involves the selection, from a wide variety of phenomena, of those one judges to have a common cause, and then the for-

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mulation of a mechanism for how that cause is to operate. The test of a good theory is its simplicity and breadth. The second step involves the selection of a specific algorithm by which to extract from the theory its implications. The test of a good algorithm is its convenience.

What underlies our confidence that physics can be structured in this way? Perhaps it is primarily our experience with present-day physical theories. In both classical and quantum physics, the theory culminates in the formulation of differential equations, while the implementation of the theory consists of selecting some algorithm—of which there are many—for solving the equations. But not all theories of physics need be of this type. We may someday in physics be confronted with a situation in which the line between the theory and its implementation is not so sharp—in which the activity of applying the theory is not so different from that of finding the theory in the first place. Specifically, we may reach a point at which there exist no algorithms whatever for applying a theory mechanically to specific circumstances. Indeed, there are suggestions that, with quantum gravity, we may already be at this point.

John Wheeler has stressed on several occasions(1,2) the relation between physical laws, computation, and logic. Here, on the occasion of his 75th birthday, we discuss certain possible formulations of, and attitudes toward, these issues. Section 2 is a review of mathematics. We briefly summarize the notion of solvability, by computer, of a mathematical problem, and the closely related notion of a computable number. Section 3 involves physics. We propose, in parallel with the notion of a computable number in mathematics, that of a measurable number in a physical theory. The question of whether there exists an algorithm for implementing a theory may then be formulated more precisely as the question of whether the measurable numbers of the theory are computable. We argue that the measurable numbers are in fact computable in the familiar theories of physics, but there is no reason why this need be the case in order that a theory have predictive power. Indeed, in some recent formulations of quantum gravity as a sum over histories, there are candidates for numbers that are measurable but not computable. In Sec. 4, we speculate as to the consequences for physics should there be accepted a theory having measurable numbers that are not computable. We suggest that such a theory should be no more unsettling to physics than has the existence of well-posed problems unsolvable by any algorithm been to mathematics.

But, in the spirit of many of John Wheeler's papers, we aim more to raise issues and stimulate discussion than to state conclusions.