On the Electromagnetic Interaction in Relativistic Quantum Mechanics

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A fundamental problem in the construction of local electromagnetic interactions in the framework of relativistic wave equations of Klein–Gordon or Dirac type is discussed, and shown to be resolved in a relativistic quantum theory of events described by functions in a Hilbert space on the manifold of space-time. The relation, abstracted from the structure of the electromagnetic current, between sequences of events, parametrized by an evolution parameter \( \tau \) ("historical time"), and the commonly accepted notion of particles is reviewed. As an illustration of these ideas, a perturbative calculation is made for photon emission from a charged two-body system in which the electromagnetic field is quantized in the usual way. The result is in essential agreement with calculations in which the charged particles are treated in the framework of nonrelativistic quantum mechanics, and provides them with a relativistic interpretation. In particular, we obtain a relativistically invariant form of the Bohr radiation condition.

1. INTRODUCTION

The original ideas of de Broglie on the structure of wave mechanics were essentially relativistic, but the subsequent, highly successful, development of quantum mechanics, including its axiomatic foundation, took place in a nonrelativistic, Galilean framework. The remarkable successes of the quantum theory in dealing with, for example, atomic spectra are, however, brought into evidence through the electromagnetic interaction, an intrin-

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sically relativistic phenomenon. The structure of the bound state levels of the Schrödinger equation with phenomenological potentials is observed by means of the radiation emitted and absorbed during transitions between these levels. Aside from the evident desirability of developing a consistent relativistic quantum theory for application to problems for which relativistic kinematics are required, and to serve as a rational basis for second quantization, the construction of a theory in which particles with a nonrelativistic description undergo electromagnetic interaction encounters logical difficulties on a microscopic level. For example, classical Hamiltonian dynamics suggest that electromagnetic interaction, for spinless particles, be introduced as a minimal gauge coupling, i.e., replacing the canonical momentum $p$ by $p - eA$, where the vector potential field $A$ is to be evaluated at the position of the particle. In its application in nonrelativistic quantum mechanics, the corresponding operator appears in conjunction with a wave function $\psi(x)$, and the potential field is evaluated at the point $x$, i.e., as $(p - eA(x))\psi(x)$. Since $|\psi(x)|^2$ has the interpretation of the probability density for the occurrence of a particle at $x$, this procedure seems to be consistent with the prescription of classical mechanics. Starting, however, with a relativistic wave equation of the Dirac or Klein–Gordon type, the same procedure does not appear to be consistent with the notion of local interaction with a charged particle. Newton and Wigner (3) have shown that the (on-shell) solutions of relativistic wave equations of this type are not local descriptions of the associated particles. The localized free particle is associated with a wave function which is spread out over a distance of the order of a de Broglie wavelength $\hbar/mc$. Locality can be restored to wave functions of this type by a Foldy–Wouthuysen transformation, but the transformed electromagnetic coupling then occurs in the equations in a very complicated way, reflecting the essential nonlocality of the coupling in the original wave equations. Although some useful estimates can be obtained in this way, it is clear that at no point has the electromagnetic interaction been introduced as a local interaction in accordance with classical Hamiltonian dynamics.

The principal difficulty in reconciling quantum mechanics and relativity lies in the role and interpretation of time. The Lorentz transformations of special relativity induce linear combinations of the time and space coordinate variables which describe the fundamental entities, events, of the relativistic world. This phenomenon implies that space and time must be dynamical variables of essentially the same type, and it is difficult for the time variable to simultaneously play the role of evolution parameter as it does in Galilean dynamics. Modern relativistic quantum field theory, which has been highly developed in recent years for quantitative applications of a primarily spectral nature, deals with this problem by recognizing the fields as dynamical variables and treats the space-time manifold as parameters.