ON THE PROBLEM OF AXISYMMETRICALLY LOADED SHELLS OF REVOLUTION WITH SMALL ELASTIC STRAINS AND ARBITRARILY LARGE AXIAL DEFLECTIONS

Hwang Chien (黄 黠)
(Shanghai University of Technology, Shanghai)
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Abstract

For the problem of axisymmetrically loaded shells of revolution with small elastic strains and arbitrarily large axial deflections, this paper suggests a group of state variables: radial displacement \( u \), axial displacement \( \omega \), angular deflection of tangent in the meridian \( \chi \), radial stress resultant \( H \) and meridional bending moment \( M_\phi \), and derives a System of First-order Nonlinear Differential Equations under global coordinate system with these variables. The Principle of Minimum Potential Energy for the problem is obtained by means of weighted residual method, and its Generalized Variational Principle by means of identified Lagrange multiplier method.

This paper also presents a Method of Variable-characteristic Nondimensionization with a scale of load parameter, which may efficiently raise the probability of success for nonlinearity calculation. The obtained Nondimensional System of Differential Equations and Nondimensional Principle of Minimum Potential Energy could be taken as the theoretical basis for the numerical computation of axisymmetrical shells with arbitrarily large deflections.

I. Introduction

In engineering, some elastic elements, undergoing especially large deformation, may be used as high accurate and sensitive transducers which are often composed of several axisymmetrically loaded shells of revolution. Under pre-buckling and pre-yield stage of small elastic strains, their angular deflections are usually finite while their axial displacement could be very large. The analysis of such elastic elements can be called the problem of axisymmetrical shells with arbitrarily large deflections and possesses the following features:

1. Very large deflections, sometimes surpass twenty times of thickness, and finite angular deflections of tangent in the meridian, sometimes reach 0.3 radian, can occur with small strains.
2. The wide range of the colatitude angle before deformation can result in large tangential displacements in the meridian.
3. The ratio of local curvature radius to thickness may be not very large, though the diameter of the shell is much larger than its thickness.
4. The generatrix of the shell of revolution may consists of several sections; and the curvature or/and the direction of tangent in the meridian may change abruptly at the joints.
5. The thickness of shells varies along the generatrix.
6. The radius of curvature varies along the generatrix.

Adopting global coordinate system, E. Reissner\textsuperscript{[1,3,12]} established two second-order differential equations of mixed method in terms of colatitude angle $\varphi$ and the function of radial displacement $rH$ as basic variables. The equations are suitable for the axisymmetrically loaded shells of revolution with large axial displacement, finite angular deflection of tangent in the meridian, small elastic strains and a wide range of original colatitude angles. They have no singularities at points where the normals to the middle surface are parallel to the axis of revolution, which are caused by using the function of shearing force $r_{e}Q_{e}$ as one of the basic variables. Thus, the theoretical basis is settled for the axisymmetrical shells with arbitrarily large deflections. R. Schmidt\textsuperscript{[2]} (1977) formulated the equations of displacement method in terms of $u$ and $\varphi$ as basic variables. L. E. Andrews\textsuperscript{[3]} (1981) presented equations similar to Reissner's equations and calculated many elastic elements with difference method.

The Reissner's and Schmidt's equations involve the variable thickness and radius of curvature along the generatrix. In second-order differential equations, this means introducing the derivatives of the thickness and the radius of curvature. Furthermore, it must be regarded as boundary and introduced transition and link conditions where the curvature and direction of tangent change abruptly. These cause some inconvenience during the calculation.

Adopting $u, w, \varphi, H$ and $M_{\varphi}$ as state variables, this paper presents a system of first-order differential equations based on the relations consistent with Reissner's ones. These state variables remain continuous everywhere along the generatrix even if the curvature and direction of tangent are not continuously changing. It is not necessary to take those abruptly changing points as boundaries; then the calculation can be carried out continuously. In the system of first-order equations, the change of the thickness and radius of curvature along the generatrix can be defined by themselves without introducing their derivatives into the equations. That is convenient for calculation.

The system of the first-order differential equations can describe the deformation properties of the shells more directly and exactly. On the basis of relations presented by Reissner\textsuperscript{[1,3]}, it is convenient for this paper to make a further consideration of the effects of curvature upon stress resultant-strain relations.

Prof. Chien Wei-zang presented the method of introducing Lagrange multipliers and identifying them\textsuperscript{[5-9]} (1964, 1979, 1983), and derived Generalized Variational Principle of thin shells with large deflection under the hypothesis that the span is much less than the radius of curvature (i.e. shallow shells)\textsuperscript{[7]} (1980). K. Washizu\textsuperscript{[10]} (1968, 1975) presented the expression of potential energy for linear problem of shells, and discussed some nonlinear terms in strain-displacement relations. Expressing the first-order nonlinear differential relations for the problem of axisymmetrical shells with arbitrarily large deflections in global coordinate system, this paper presents its Minimum Potential Energy Principle by means of weighted residual method and its Generalized Variational Principle by means of identified Lagrange multiplier method.

This paper also presents a Method of Variable-characteristic Nondimensionization by using load parameter as denominators of nondimensionizing factors. When the load vanishes, singularity appears. In this paper, it is confirmed that at the singular point the nondimensional solutions have finite limits which are just the linear simplified solutions degenerated from the original nonlinear equations in nondimensional form. So the singular point is a removable singularity.

Using this nondimensionization, the valid values of axial deflection obtained by perturbation method surpass twenty times of shell thickness, breaking the traditional opinion that perturbation