FUNCTION OF REGION

Ho Chong (何仲)
(Institute of Scientific Sinica Information, Chongqing)

(Received Nov. 9, 1983 Communicated by Chien Wei-zang)

Abstract

The purpose of this paper is to extend points function and interval functions theoretics to an arbitrary region. For this, the new theory, the contraction of a region, and the retraction of a region; the extension of a region, and the kernel-preserving extension of a region are established by the author. Starting from these concepts, the new definitions of a region function is given. And a kernel (i.e. fixed point) of a region function is connected with a stable centre of defining region of such a region function. Thereby, the region theoretics and algorithms are established.

In applications, to find a stable centre of a region, the author has utilized the measure theoretics of matrice defined by Hartfiel[1] and other authors. The measure problems of coefficient matrice of system of equations of linear algebra associated with some region are discussed.

I. Introduction

The function discussed for a long time in the past is points function, J. C. Burkill was the first to establish the intervals function, and also the first to discuss its properties; and he has presented some fundament concepts in 1923.

Up to now, whether the points function or the intervals function, the derivatives of these functions are defined to utilize the classical method, namely, the mapping is used.

In order to extend the points function and intervals function, and the properties and methods related to them to an arbitrary nonempty region, the author will utilize the change of a region (or, contraction, or, extension, or, making any kernel-preserving transformation) to define the region function, namely

\[ F(x) \subseteq B(R), \quad B(R) \subseteq F(x), \quad \forall x \in B(R) \]  

The author has used some of the basic concepts, for the topology of points set[2] and the topology of algebra[3], in the process of defining this theoretics. However, the concepts are different from these theoretics.

In this paper, on the one hand, the author tried to establish some basic theoretics of a region and a region function; on the other hand, the author also tried to establish a region algorithm, using these basic theoretics and methods as a tool. These methods, for defining the differential of a region function and solving the generalized differential equations, generalized integral equations, generalized integrato-differential equations etc., are specially effective; they are largely to open the convenient doors for the further development of geometry, physics, especially linear geometry,
nonlinear geometry, differential geometry etc.

This paper attaches importance to the general and abstract algorithms of values of a region, namely, the method of stable centre of a region. This method is mainly based on the inverse of P. Huard's ideas[1] and apply the Ehrmann's and Schröder's[2] fixed point theorems to the region, namely, from the region to a subregion (i.e. stable centre) of such a region, or, from the multivalues to a single-value. This is one of the prime ideas of the paper.

This paper is divided into three parts. The first part is to give the symbols and definitions used throughout the paper; the second part gives some basic lemmas, theorems, and corollaries in detail to prove some results; the third part gives the computing methods of values of a region by using the defined basic theoretics of this paper and passing to apply E. Spanier's[6] methods of algebraic topology and D. J. Hartfiel's[7] theoretics of matrix measure to the region.

II. Symbols and Definitions

For the requirement of this paper, here, we give some of the symbols and definition as follows:

We denote an arbitrary nonempty region (bounded, or unbounded; simply connected, or complex connected) by \( R \); denote a collection of all \( R \) by \( B(R) \); denote the variable of an arbitrary \( n \)-dimensional region by \( x := (x_1, x_2, ..., x_n) \) (or, \( x := \{ x_i \} \) \( i = 1(1)n \)); denote the region function defined over the region \( B(R) \), by \( F(X) \); denote the closed region by \( \overline{B(R)} \) (or, \( \overline{R} \)); denote the arbitrary open region by \( B(R) \) (or, \( R \)); denote a stable centre of a region \( B(R) \) (or, \( R \)) by \( B(\text{go}) \) (or, \( R_0 \)); denote the value of a region \( B(R) \) at a stable centre \( B(\text{go}) \) (or, \( R_0 \)) by \( F(B(R)) \) (or, \( F(R) \)); denote a point at a stable centre of a region \( B(R) \), by \( x_0 \); denote the value of a region function \( F(x) \) at a point \( x_0 \) on a stable centre \( B(R_0) \) of its defining region \( B(R) \) by \( F(x_0) \).

**Definition 1** A subregion \( B(R_0) \) of a region \( B(R) \) is said to be a stable centre (or, approximate stable centre) of such a region \( B(R) \), if the following conditions are always true.

1) \( B(R_0) \) is always stable, or approximate stable, when region \( B(R) \) is doing the arbitrary run, or the arbitrary transformation;

2) A kernel \( x_0 \) (or, fixed point of a mapping, or transformation) of a region function \( F(x) \) defined over such a region \( B(R) \), always lies on a subregion \( B(R_0) \) of region \( B(R) \).

Then, \( B(R_0) \) is absolute stable centre for region \( B(R) \), if the \( F(x_0) = x_0 \) and \( x_0 \in B(R_0) \) is always true.

Then, \( B(R_0) \) is relative stable centre for region \( B(R) \), if, \( x_0 \in F(x) \), \( \forall x \in B(R) \), and \( x_0 \in B(R_0) \) is always true.

Then, \( B(R_0) \) is doing an arbitrary kernel-preserving transformation, if, \( B(R) \supseteq F(x) \), \( \forall x \in B(R) \), is always true.

**Definition 2** Suppose that \( B(R) \) is any nonempty region. If \( G(x) \subseteq B(R) \), \( \forall x \in B(R) \) is always true, \( G \) is a contraction of region \( B(R) \). If there is a point \( x_0 \) belonging to \( G(x) \), \( G \) is a retraction of region \( B(R) \). Inversely, if \( B(R) \subseteq G(x) \), \( \forall x \in B(R) \) is always true, \( G \) is an extension of region \( B(R) \). In the same way, if there is a point \( x_0 \) belonging to \( G(x) \), \( G \) is known as a kernel-preserving extension of region \( B(R) \). If \( G(x) \subseteq B(R) \), \( \forall x \in B(R) \) is always true, and there exists a point \( x_0 \) belonging to \( G(x) \), \( G \) is a retraction of itself from region \( B(R) \); similarly, if \( B(R) \subseteq G(x) \), \( \forall x \in B(R) \), and there exists a determinate point \( x_0 \), such that \( x_0 \in G(x) \), is always true, then, \( G \) is a kernel of itself preserving extension from region \( B(R) \).

If \( F(x) \subseteq B(R) \), \( B(R) \subseteq F(x) \), \( \forall x \in B(R) \) are always true, \( F(x) \) is a region function defined over such a region \( B(R) \).

**Definition 3** Suppose that a number of defining regions of the region functions are always