THE THEORY OF FUNCTIONS OF A COMPLEX VARIABLE
UNDER DIRAC—PAULI REPRESENTATION AND ITS
APPLICATION IN FLUID DYNAMICS (I)

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Abstract

In this paper:

(A) We cast aside the traditional quaternion theory and build up the theory of functions of a complex variable under Dirac-Pauli representation. Thus the multivariate and multidimensional problems become rather simple problems.

(B) We simplify the Navier-Stokes equation of incompressible viscous fluid dynamics and the equations group of isentropic aerodynamics by theory of functions of a complex variable under Dirac-Pauli representation. And the above-equations, as central problems of fluid dynamics, are classified as the nonlinear equation with only one complex unknown function.

So the changes are the Great Pole, and give birth to the Two Bearings; the Two Bearings give birth to the Four Quadrants, and the Four Quadrants give birth to the Eight Diagrams.

—Commentaries on Changes, Copula (I)

I. Introduction

As everyone knows, the lead of Airy stress function in plane problem of static elasticity theory, the lead of flow function in plane flow or axial symmetry flow of incompressible fluid, and the lead of velocity potential in irrotational flow problem is very convenient. The present problem is whether this method of defining the solutions of some mechanics problems by only one unknown function can be popularized to three-dimensional problems (or four-dimensional spacetime) or not? This is the first problem.

Secondly, it is known to all that the success of traditional theory of functions of a complex variable is brilliant and tremendous. But, it is simply efficacious on two-dimensional problems (or three-dimensional spacetime). Can this simple and convenient method of expression by complex number form we be applied to three-dimensional problems (or four-dimensional spacetime)? If we can, what kind of form it has?

Thirdly, it is known to everyone who works in inverse scattering transformation and soliton theory that up till now the theory of these problems is mostly on one-dimensional problems (or two-dimensional spacetime) and only one unknown function. If we solve the mechanics problems
for three-dimensional space or four-dimensional spacetime by inverse scattering transformation and soliton theory, there are only two methods, to popularize the present theory to multidimensional space\textsuperscript{10–11}, and to turn mechanics problems into the form on only one unknown function and few independent variable. We can equate the first method with the second one on degree of mathematical difficulty. And, the second method directs bearing on the two preceding problems.

From the standpoint of inverse scattering transformation and soliton theory, in all the difficulties of nonlinear mechanics or physics problems, chiefly among them are dimension of spacetime and number of unknown functions. By experience and outcome of this paper, the difficulties of problems roughly direct ratio to $\log_2 n$, where $n$ is dimension of space [or $(n + 1)$-dimensional spacetime] and number of unknown functions.

Up to now we solve the three above problems only by one mathematical method, that is, the so-called “quaternion” theory\textsuperscript{12–14}. We already know that the “ternary numbers” don’t exist, and at last the vector replaces it\textsuperscript{15}. In papers \textsuperscript{15} and \textsuperscript{16} we have proved the inexpansibility of number field in principle, will not keep up all basic operational rule of arithmetic operation at the same time. But if we give up some operational rule, then we can prove the expansibility of number field. The quaternion theory is built on the basis of giving up commutation law of multiplication. Moreover, on the biquaternion theory we still give up more operational rules.

W. R. Hamilton was the first man to introduce quaternion theory into mathematics (1834). Since 1833 he began to study quaternion theory of his own. His achievements in scientific research crystallized in two books\textsuperscript{17–18}. Soon afterwards, F. Klein\textsuperscript{19} et al. also did some work. But up to now the quaternion theory has not found practical application of great value, and remains on mathematical imperial crown in ivory tower\textsuperscript{20–21} as the example of formal mathematical model of linear algebra. In abroad someone applied the quaternion theory to orientation problems of rigid body\textsuperscript{22}, but not simple in form, and the achievements in scientific research is limited.

In this paper we cast aside the traditional quaternion theory and build up the theory of functions of a complex variable under Dirac-Pauli representation. From this paper the theory of functions of a complex variable under Dirac-Pauli representation can replace quaternion theory as independent mathematical course. Its calculus is simple, and its form is satisfactory.

In this paper we take fluid dynamics as touchstone of the theory of functions of a complex variable under Dirac-Pauli representation. On the one hand in fluid dynamics the independent variable and unknown functions have a large number of difficulties for the solution; on the other hand we can turn some of the primary equations of elasticity and plasticity into Schrödinger equation or Dirac equation\textsuperscript{10–11, 23–25} that the solution is not difficult any longer. Naturally, the methods of this paper can also enjoy the application in multidimensional elasticity and plasticity.

For little involvement of the law of thermodynamics, we take Navier-Stokes equation of incompressible viscous fluid dynamics and the equations group of isentropic aerodynamics as the target of our study in this paper. The solutions of the first equation is the key problem of fluid dynamics\textsuperscript{26}, and at the same time the first equation must be satisfied by the instant parameters of the turbance. The latter is the other key problem, that is second only to the first.

The dummy index is the summation depended on Einstein convention in this paper.

II. The Theory of Functions of a Complex Variable under Dirac-Pauli Representation

We can call the traditional theory of functions of a complex variable “the theory of functions of