PERTURBATION INITIAL PARAMETER METHOD FOR SOLVING
THE GEOMETRICAL NONLINEAR PROBLEM OF
AXISYMMETRICAL SHELLS

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Abstract

In the previous paper[7], the author presented a System of First-Order Differential Equations for the problem of axisymmetrically loaded shells of revolution with small elastic strains and arbitrarily large axial deflections, and a Method of Variable-Characteristic Nondimensionization with a Scale of Load Parameter. On this basis, by taking the weighted root-mean-square deviation of angular deflection from linearity as perturbation parameter, this paper presents a perturbation system of nondimensional differential equations for the problem, thus transforms the geometrical nonlinear problem into several linear problems. This paper calculates these linear problems by means of the initial parameter method of numerical integration. The numerical results agree quite well with the experiments[4].

I. Introduction

Based on linear analytic solutions, the perturbation method was brought into positive plays for solving nonlinear problems of plates and shells[11-3]. In pace with the progress of computers, plenty of effective numerical methods appear. In order to solve the linear problems of axisymmetrical shells, for instance, R. F. Dressier and the others[4] (1957) treated corrugated diaphragms by Runge-Kutta method; the author[5] (1982) calculated bellows by means of Gill's method.

This paper tries to combine perturbation method with initial parameter method of numerical integration so as to solve the problem of axisymmetrical shells with arbitrarily large deflections. This paper expands the system of nonlinear first-order differential equations developed by the author[7], by adopting global coordinate system with the radial displacement $u$, axial displacement $u$, the angular deflection of tangent in the meridian $\chi$, the radial stress resultant $H$ and the meridional bending moment $M_\phi$, as a group of state variables and choosing the weighted root-mean-square deviation of nondimensional angular deflection from linearity as perturbation parameter. Thus, this paper obtains several systems of linear equations. By means of Gill's method[6], numerical integrations are made continuously along the meridian of rotatory shells, of which the curvature or the direction of tangent in the meridian may change abruptly. Owing to the use of the method of variable-characteristic nondimensionization with a scale of load parameter, we extend the valid range of perturbation solutions and obtain numerical results agree with experiments.

II. Nondimensional Perturbation Differential Equations

In paper [7], the author presented a system of first-order differential equations for the problem of axisymmetrical shells with arbitrarily large deflection as follows.
\[
\begin{align*}
1 \frac{du}{d\psi} &= \varepsilon_\varphi \cos \varphi + \cos \varphi - \cos \varphi_0 \quad (2.1) \\
1 \frac{dw}{d\psi} &= -\varepsilon_\varphi \sin \varphi - (\sin \varphi - \sin \varphi_0) \quad (2.2) \\
1 \frac{d\chi}{d\psi} &= -\frac{12(1-\nu^2)}{Et^3} M_\varphi - \nu (\sin \varphi - \sin \varphi_0) / r + \Gamma \varepsilon_\varphi \quad (2.3) \\
1 \frac{dH}{d\psi} &= -\left\{ H \cos \varphi + q^* r \sin \varphi - \frac{Et}{1-\nu^2} \left[ \nu \varepsilon_\varphi + (2-\omega) u/r + \frac{t^2}{12} \Gamma (\sin \varphi - \sin \varphi_0) / r \right] \right\} / r \\
1 \frac{dM_\varphi}{d\psi} &= -\left\{ (1-\nu) M_\varphi + \frac{Et^3}{12} \left[ (\sin \varphi - \sin \varphi_0) / r + \Gamma \left( \frac{u}{r} + \nu \varepsilon_\varphi \right) / (1-\nu^2) \right] \right\} \cos \varphi / r \\
&\quad + \frac{1}{2} \left( \frac{P^*}{\pi r} + q^* r \right) \cos \varphi - H \sin \varphi \quad (2.4)
\end{align*}
\]

where
\[
\begin{align*}
\varepsilon_\varphi &= \left\{ \frac{1-\nu^2}{Et} \left[ \frac{1}{2} \left( \frac{P^*}{\pi r} + q^* r \right) \sin \varphi + H \cos \varphi - \Gamma M_\varphi \right] \\
&\quad - \frac{\nu}{r} \left( \frac{u}{r} + \frac{t^2}{12} \Gamma (\sin \varphi - \sin \varphi_0) \right) \right\} / \omega \\
\Gamma &= \frac{1}{r_1} - \frac{1}{r_2} = \frac{1}{A} \frac{d\varphi_0}{d\psi} - \sin \varphi_0 / r, \quad \omega = 1 + \frac{t^2}{12} \Gamma \sin \varphi_0 / r, \quad A = \frac{ds}{d\psi}
\end{align*}
\]

In the above expressions, \( \varphi \) is a basic parameter describing the distribution of all variables along the generatrix; \( r \) the radial coordinate of a point on the undeformed middle surface; \( \varphi_0 \) the original colatitude angle, formed by the axis of revolution and the normal to the undeformed middle surface; \( \varphi \) the colatitude angle after deformation; \( u, w \) radial and axial displacements; \( \chi \) angular deflection of tangent in the meridian. \( \chi = \varphi - \varphi_0 \); \( H \) radial stress resultant; \( M_\varphi \) meridional bending moment; \( E, \nu \) Young's modulus and Poisson's ratio; \( t \) thickness of the shell; \( r_1, r_2 \) radii of principal curvatures of the middle surface; \( ds \) infinitesimal arc length of the meridian; \( P^* \) concentrated axial load at internal rim or the center of the shell; \( q^* \) uniformly distributed normal pressure on the wall of the shell.

To solve the problem of large deflections caused by compound load, first we introduce a load parameter \( \overline{q} \) defined by
\[
\overline{q} = \max \{ q^*, \ P^*/0.25\pi D^3 \} \quad (2.6)
\]
where \( D \) is the outer diameter of the shell. The process with a constant ratio between the concentrated axial force and uniform pressure can be taken as elementary loading process in calculation.

By using the Method of Variable-Characteristic Nondimensionization with a Scale of Load Parameter presented by the author in paper [7], we introduce the following nondimensional variables: