Generalized symmetry and its application to 3D shape generation

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A new method for easily and rapidly generating three-dimensional shapes from two-dimensional line-drawings is presented. This method is based on the generalized symmetry constraint. Generalized symmetry is an extended concept of three-dimensional symmetry and its axis is a 3D smooth curve. This paper first develops the definition and constraint of generalized symmetry, and then describes an algorithm which generates the three-dimensional shape of an object from its line-drawing. The generation algorithm is extended to generate generalized cylindrical objects from line-drawings. Several experiments by computer simulation verify that the algorithm can generate three-dimensional shapes from line-drawings.

Key words: Modelling – Shape recovering – Computer vision – Man-machine interface – Symmetry
pose a generation method for objects which satisfy the constraints of "generalized symmetry". Generalized symmetry is a property of a pair of three-dimensional curves. It is an extended concept of planar-symmetry, where the property of its symmetry axis is extended to include three-dimensional curves. Generalized symmetry is a useful property for describing objects, since there are a lot of objects in nature and industry whose surface boundaries are generalized symmetrical curves. Moreover, it is possible to extend this method to the generation of objects represented by the generalized cylinder which was introduced to Computer Vision to extensively represent three-dimensional volumes (Agin and Binford 1976). This paper discusses this, and it is shown that various kinds of bulky objects can be created from line-drawings using a slightly modified property of generalized symmetry.

This paper develops (1) a generalized symmetry, (2) a generation algorithm based on the generalized symmetry constraint, and (3) applications for generalized cylindrical object modelling.

2 Principle of shape generation

2.1 Generalized symmetry

Figure 1a shows a symmetrical line-drawing, which is planar and symmetrical about a straight symmetry axis. Figure 1b is not strictly symmetrical. However, it can be regarded as the projection of a real symmetrical shape mapped in three-dimensional space. The property of this line-drawing is called skewed symmetry. Skewed symmetry was proposed by Kanade (1980). He proposed a method of recovering the three-dimensional shape based on the skewed symmetry constraint.

On the other hand, the line-drawing in Fig. 1c is neither symmetrical nor skewed symmetrical. However, one can understand that it depicts a leaf, and can recognize it as a symmetrical object whose symmetry axis is a curve in three-dimensional space. We named this kind of symmetry as generalized symmetry. Generalized symmetry is a property of paired three-dimensional smooth curves, where their symmetry axis is also a three-dimensional smooth curve. The pair of three-dimensional curves twist around their generalized symmetry axis. Considering Fig. 1c, the leaf is recognizable even thought its generated 3D shape has no thickness.

Many objects in the real world have a similar property. Thus, the generation method based on generalized symmetry is expected to be a useful technique and provide a basis for regenerating three-dimensional objects from two-dimensional drawings.

2.2 Definition of generalized symmetry

The mathematical definition of generalized symmetry is described as follows. In Fig. 2, $B_1$, $B_2$, and $C$ are three-dimensional smooth curves, which are first-order differentiable. $P$ is any point on curve $C$. $I$ is a unit tangent vector at $P$ on $C$. Plane $S$ is perpendicular to vector $I$ and passes through point $P$. $P_1$ is the intersection of $B_1$ and $S$, and $P_2$ lies on $B_2$ and $S$. 

![Fig. 1a–c. Three types of symmetry. a Real symmetry. b Skewed symmetry. c Generalized symmetry](image-url)