Generalized symmetry and its application to 3D shape generation

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A new method for easily and rapidly generating three-dimensional shapes from two-dimensional line-drawings is presented. This method is based on the generalized symmetry constraint. Generalized symmetry is an extended concept of three-dimensional symmetry and its axis is a 3D smooth curve. This paper first develops the definition and constraint of generalized symmetry, and then describes an algorithm which generates the three-dimensional shape of an object from its line-drawing. The generation algorithm is extended to generate generalized cylindrical objects from line-drawings. Several experiments by computer simulation verify that the algorithm can generate three-dimensional shapes from line-drawings.

Key words: Modelling - Shape recovering - Computer vision - Man-machine interface - Symmetry

With the recent progress in computer graphics technology, images of very complex scenes can be synthesized. However, shape modelling still remains a great problem that demands a lot of effort from graphic designers.

Objects are usually designed with one of two typical modelling methods. The first creates a 3D shape from more than two plans of an object. Each plan is a projection of the object at different projection angles. The second constructs 3D shapes by combining or intersecting simple parts or primitive shapes. In these methods, designers must estimate and input a large number of positions of surface points or primitives using either data-tablet or keyboard. Furthermore, this task involves a lot of trial and error. This is one of the serious problems that limit productivity and restrict the application areas of computer graphics.

To improve modelling systems, several useful techniques have been proposed. One of them is the modification method. Both Barr and Sederberg proposed methods of deforming solid geometric models (Barr 1984; Sederberg 1986). Pentland proposed a method which constructs natural forms with deformed primitive shapes (Pentland 1986). Another method creates 3D shapes from their cross sections. Woodward proposed this method for interpolating between cross sections (Woodward 1987). Finally, generation rule methods for specific natural objects, such as trees, mountains, and waves, have been proposed by Smith, Oppenheimer, Reeves, and others (Smith 1984; Oppenheimer 1986; Reeves 1983).

These methods are useful to design various 3D shapes. However, with only these methods, it does not seem possible to establish user-friendly modelling systems, since they can manipulate only limited classes of 3D shapes. A variety of methods are necessary to create various classes of realistic 3D shapes.

One of the most useful approaches, we believe, to realize a user-friendly man-machine interface is to generate the 3D shape of an object from freehand line-drawings. This is because line-drawings are often used to represent 3D shapes and everyone can make line-drawings.

Kanade has proposed a recovery principle for a symmetrical planar shape when it is mapped into three-dimensional space (Kanade 1980). This method recovers the original shape from its projected line-drawing when the shape consisted of symmetrical planar surfaces. However, this method cannot be applied to objects with curved surfaces. In order to recover a more general class of shapes, we pro-
pose a generation method for objects which satisfy
the constraints of "generalized symmetry".
Generalized symmetry is a property of a pair of
three-dimensional curves. It is an extended concept
of planar-symmetry, where the property of its sym-
metry axis is extended to include three-dimensional
curves. Generalized symmetry is a useful property
for describing objects, since there are a lot of ob-
jects in nature and industry whose surface bound-
aries are generalized symmetrical curves.
Moreover, it is possible to extend this method to
the generation of objects represented by the gener-
alized cylinder which was introduced to Computer
Vision to extensively represent three-dimensional
volumes (Agin and Binford 1976). This paper dis-

cusses this, and it is shown that various kinds of
bulky objects can be created from line-drawings
using a slightly modified property of generalized
symmetry.
This paper develops (1) a generalized symmetry,
(2) a generation algorithm based on the generalized
symmetry constraint, and (3) applications for gen-
eralized cylindrical object modelling.

2 Principle of shape generation

2.1 Generalized symmetry

Figure 1a shows a symmetrical line-drawing,
which is planar and symmetrical about a straight
symmetry axis. Figure 1b is not strictly symmetri-
cal. However, it can be regarded as the projection
of a real symmetrical shape mapped in three-di-
mensional space. The property of this line-drawing
is called skewed symmetry. Skewed symmetry was
proposed by Kanade (1980). He proposed a meth-

od of recovering the three-dimensional shape based
on the skewed symmetry constraint.
On the other hand, the line-drawing in Fig. 1c is
neither symmetrical nor skewed symmetrical.
However, one can understand that it depicts a leaf,
and can recognize it as a symmetrical object whose
symmetry axis is a curve in three-dimensional
space. We named this kind of symmetry as general-
ized symmetry. Generalized symmetry is a property
of paired three-dimensional smooth curves, where
their symmetry axis is also a three-dimensional
smooth curve. The pair of three-dimensional curves
twist around their generalized symmetry axis.
Considering Fig. 1c, the leaf is recognizable even
thought its generated 3D shape has no thickness.

Many objects in the real world have a similar prop-
erty. Thus, the generation method based on gener-
alized symmetry is expected to be a useful tech-
nique and provide a basis for regenerating three-
dimensional objects from two-dimensional draw-

ings.

2.2 Definition of generalized symmetry

The mathematical definition of generalized symme-
try is described as follows. In Fig. 2, $B_1$, $B_2$, and
C are three-dimensional smooth curves, which are
first-order differentiable. $P$ is any point on curve
$C$. $I$ is a unit tangent vector at $P$ on $C$. Plane $S$
is perpendicular to vector $I$ and passes through
point $P$. $P_1$ is the intersection of $B_1$ and $S$, and
$P_2$ lies on $B_2$ and $S$. 

Fig. 1a-c. Three types of
symmetry. a Real symmetry.
b Skewed symmetry.
c Generalized symmetry