Generalized symmetry and its application to 3D shape generation

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A new method for easily and rapidly generating three-dimensional shapes from two-dimensional line-drawings is presented. This method is based on the generalized symmetry constraint. Generalized symmetry is an extended concept of three-dimensional symmetry and its axis is a 3D smooth curve. This paper first develops the definition and constraint of generalized symmetry, and then describes an algorithm which generates the three-dimensional shape of an object from its line-drawing. The generation algorithm is extended to generate generalized cylindrical objects from line-drawings. Several experiments by computer simulation verify that the algorithm can generate three-dimensional shapes from line-drawings.

Key words: Modelling – Shape recovering – Computer vision – Man-machine interface – Symmetry

With the recent progress in computer graphics technology, images of very complex scenes can be synthesized. However, shape modelling still remains a great problem that demands a lot of effort from graphic designers.

Objects are usually designed with one of two typical modelling methods. The first creates a 3D shape from more than two plans of an object. Each plan is a projection of the object at different projection angles. The second constructs 3D shapes by combining or intersecting simple parts or primitive shapes. In these methods, designers must estimate and input a large number of positions of surface points or primitives using either data-tablet or keyboard. Furthermore, this task involves a lot of trial and error. This is one of the serious problems that limit productivity and restrict the application areas of computer graphics.

To improve modelling systems, several useful techniques have been proposed. One of them is the modification method. Both Barr and Sederberg proposed methods of deforming solid geometric models (Barr 1984; Sederberg 1986). Pentland proposed a method which constructs natural forms with deformed primitive shapes (Pentland 1986). Another method creates 3D shapes from their cross sections. Woodward proposed this method for interpolating between cross sections (Woodward 1987). Finally, generation rule methods for specific natural objects, such as trees, mountains, and waves, have been proposed by Smith, Oppenheimer, Reeves, and others (Smith 1984; Oppenheimer 1986; Reeves 1983).

These methods are useful to design various 3D shapes. However, with only these methods, it does not seem possible to establish user-friendly modelling systems, since they can manipulate only limited classes of 3D shapes. A variety of methods are necessary to create various classes of realistic 3D shapes.

One of the most useful approaches, we believe, to realize a user-friendly man-machine interface is to generate the 3D shape of an object from freehand line-drawings. This is because line-drawings are often used to represent 3D shapes and everyone can make line-drawings.

Kanade has proposed a recovery principle for a symmetrical planar shape when it is mapped into three-dimensional space (Kanade 1980). This method recovers the original shape from its projected line-drawing when the shape consisted of symmetrical planar surfaces. However, this method cannot be applied to objects with curved surfaces. In order to recover a more general class of shapes, we pro-
pose a generation method for objects which satisfy the constraints of "generalized symmetry".

Generalized symmetry is a property of a pair of three-dimensional curves. It is an extended concept of planar-symmetry, where the property of its symmetry axis is extended to include three-dimensional curves. Generalized symmetry is a useful property for describing objects, since there are a lot of objects in nature and industry whose surface boundaries are generalized symmetrical curves.

Moreover, it is possible to extend this method to the generation of objects represented by the generalized cylinder which was introduced to Computer Vision to extensively represent three-dimensional volumes (Agin and Binford 1976). This paper discusses this, and it is shown that various kinds of bulky objects can be created from line-drawings using a slightly modified property of generalized symmetry.

This paper develops (1) a generalized symmetry, (2) a generation algorithm based on the generalized symmetry constraint, and (3) applications for generalized cylindrical object modelling.

2 Principle of shape generation

2.1 Generalized symmetry

Figure 1a shows a symmetrical line-drawing, which is planar and symmetrical about a straight symmetry axis. Figure 1b is not strictly symmetrical. However, it can be regarded as the projection of a real symmetrical shape mapped in three-dimensional space. The property of this line-drawing is called skewed symmetry. Skewed symmetry was proposed by Kanade (1980). He proposed a method of recovering the three-dimensional shape based on the skewed symmetry constraint.

On the other hand, the line-drawing in Fig. 1c is neither symmetrical nor skewed symmetrical. However, one can understand that it depicts a leaf, and can recognize it as a symmetrical object whose symmetry axis is a curve in three-dimensional space. We named this kind of symmetry as generalized symmetry. Generalized symmetry is a property of paired three-dimensional smooth curves, where their symmetry axis is also a three-dimensional smooth curve. The pair of three-dimensional curves twist around their generalized symmetry axis. Considering Fig. 1c, the leaf is recognizable even thought its generated 3D shape has no thickness.

Many objects in the real world have a similar property. Thus, the generation method based on generalized symmetry is expected to be a useful technique and provide a basis for regenerating three-dimensional objects from two-dimensional drawings.

2.2 Definition of generalized symmetry

The mathematical definition of generalized symmetry is described as follows. In Fig. 2, $B_1$, $B_2$, and $C$ are three-dimensional smooth curves, which are first-order differentiable. $P$ is any point on curve $C$. $I$ is a unit tangent vector at $P$ on $C$. Plane $S$ is perpendicular to vector $I$ and passes through point $P$. $P_1$ is the intersection of $B_1$ and $S$, and $P_2$ lies on $B_2$ and $S$. 