Abstract. Most combination equations for evaporation rely on a linear expansion of the saturation vapor-pressure curve around the air temperature. Because the temperature at the surface may differ from this temperature by several degrees, and because the saturation vapor-pressure curve is nonlinear, this approximation leads to a certain degree of error in those evaporation equations. It is possible, however, to introduce higher-order polynomial approximations for the saturation vapor-pressure curve and to derive a family of explicit equations for evaporation, having any desired degree of accuracy. Under the linear approximation, the new family of equations for evaporation reduces, in particular cases, to the combination equations of H. L. Penman (Natural evaporation from open water, bare soil and grass, Proc. R. Soc. London, Ser. A 193, 120-145, 1948) and of subsequent workers. Comparison of the linear and quadratic approximations leads to a simple approximate expression for the error associated with the linear case. Equations based on the conventional linear approximation consistently underestimate evaporation, sometimes by a substantial amount.

Introduction

The 'combination method' for estimation of evaporation refers to the simultaneous solution of the equations of surface energy balance and turbulent transport of heat and water vapor, sometimes with allowance for internal plant resistance to transpiration. This approach was initiated by Penman (1948), who used several approximations to arrive at an explicit 'combination equation' for the evaporation rate as a function of several easily measured quantities. The empirical components and some of the physical limitations of Penman's original equation were later removed by him and a series of other investigators (Penman and Schofield, 1951; Businger, 1956; Tanner and Pelton, 1960; Monteith, 1965; Van Bavel, 1966; Thom, 1972; Thom and Oliver, 1972). Brutsaert (1982) provides a sound overview.

One approximation (in fact the critical step), introduced by Penman to obtain an explicit evaporation equation, has persisted to this day. To eliminate the effective surface temperature from the system of equations, he introduced a linear relation between temperature and saturation vapor pressure, with the slope $\Delta$ of the relation determined at the air temperature. An exact solution would instead use the chord-slope evaluated between the air temperature and the surface temperature. The latter is not known a priori, so the approximation is necessary to avoid iteration.

Some investigators have made estimates of the error induced by ignoring the temperature difference between air and underlying surface in applications of Penman's combination equation. Tanner and Pelton (1960) used data for Madison, Wisconsin, to show that the relative error in evaporation rate was less than one-tenth of the relative error in $\Delta$ for March through September. Temperature
measurements made for 8 days above and at an alfalfa-brome surface suggested that the relative error of A averaged about 10%, so they concluded that relative error in computed evaporation rate would be negligible.

Van Bavel (1966) considered three widely differing climatic situations, and leaf-air temperature differences reported in the literature, in an assessment of the error induced by ignoring the temperature difference. He acknowledged that the analysis was artificial, because the temperature differences were specified independently of the other climatic conditions. He concluded that the approximation was an adequate working assumption, because the errors were less than instrumental errors, “and the utility of an iterative solution to the problem” was then ‘questionable’.

Slatyer and McIlroy (1961, Chapter 3, p. 70) stated that the approximation led to as much as 10% error in calculated evaporation rates, citing unpublished data of McIlroy.

Paw U and Gao (1988) considered hypothetical situations in which the conventional linearization of the saturation vapor pressure curve led to relative errors in evaporation as high as 10% or more. Almost all the errors were negative.

The possibility of iterating for a mathematically exact solution of the combination problem has been recognized. This is the approach taken by Budyko (1951), and several others. However, this approach is not universally applied, despite the obvious benefits in accuracy. Undoubtedly the operational simplicity of a closed-form, explicit equation for evaporation is one of the main reasons for this, though this advantage is probably more apparent than real; the iteration procedure itself is computationally trivial. Additionally, there is a certain theoretical appeal to an expression that directly reveals the dependence of the evaporation rate upon the various related variables. It has also been argued (somewhat unconvincingly) that if the measurement errors are as large as the errors introduced by the mathematical approximation, then there is no reason to improve the computations.

Paw U and Gao (1988) have recently presented a method that has higher accuracy than the usual linearization approach and does not require iteration. They represented the saturation vapor-pressure curve using a second-order Taylor series, and arrived at a quadratic equation for the evaporation rate, with coefficients dependent on the various atmospheric and surface factors. Taking their work one step further, one can obtain an explicit, second-order equation for evaporation. Paw U and Gao also presented a quartic solution procedure corresponding to a fourth-order approximation to the saturation vapor pressure curve. However, the explicit evaporation equation that could result from this method is extremely complex and would have little theoretical appeal for purposes other than computation.

The current work was performed independently of that of Paw U and Gao (1988). It is generally similar in thrust, but results in a family of evaporation equations that are considerably simpler in form, while yielding any desired degree of accuracy.