SOME ASPECTS OF MATHEMATICAL STATISTICS
AS APPLIED TO NONISOTHERMAL KINETICS V.

Comparison of the Amount of Information Obtained in Traditional and Nontraditional Approaches

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The paper gives a quantitative comparison of two methodological approaches to the solution of the inverse kinetic problem: the traditional approach and the nontraditional approach suggested by the authors. It is shown that the amount of information (in the sense of Shannon) obtained within the scope of the nontraditional approach is always greater than that obtained with the use of the traditional approach.

In the previous Part [1] considering the methodological aspects of a formal description of heterogeneous processes, two approaches to the ambiguous solution of the inverse kinetic problem, traditional and nontraditional, were emphasized.

The former is based on the principle of an unambiguous description; the latter relies on the principle of complementarity. The salient feature of the nontraditional approach is the simultaneous application of several kinetic functions to describe the process, which allows a description yielding more information. The present work seeks to give a quantitative estimation of the merits of the nontraditional methodology, as far as the information obtained is concerned, in solving the inverse kinetic problem.

According to Lindley [2], the information obtained in the experiment corresponds to the variations of the entropy of a posteriori distribution $H(p^*)$ as against the entropy of a priori distribution $H(p^0)$:

$$ I = H(p^0) - H(p^*) $$

(1)

The entropy of a discrete distribution is estimated through the Shannon formula [3] as

$$ H(p) = - \sum_{i=1}^{L} p_i \log p_i $$

(2)
where $p_i$ is the probability of the $i$-th event. As the experimental data produce information only within the scope of the appropriate models [4], we shall give some elucidation concerning the models and information under consideration.

Any quantitative method of interpreting the experimental data generates its own probability distributions. The method of solution of the inverse problem is no exception in this sense. As concerns our classification into traditional and nontraditional methods of solution of the inverse kinetic problem [1], two appropriate solution models may be singled out. Mathematically, they differ in generating different kinds of a posteriori distribution $p^*$, which will be shown below. It should be noted that the information produced by these models is that obtained from the selection of the kinetic functions used to describe the experimental data. The principle of selection is determined by the model of solution of the inverse kinetic problem, i.e. by the methodology used.

A set of about twenty kinetic functions is usually employed to describe heterogeneous processes. Then, in terms of (1), the $i$-th event is the description of the experimental data by the $i$-th kinetic function; $p_i$ is the description probability for the $i$-th function; $L$ is the number of functions used for the description. In the case when there is no a priori information on the process corresponding to some kinetic function, all of them are equally probable, which is consistent with uniform a priori distribution $p^*$ (Fig. 1). The entropy of a uniform distribution in formula (2) is $\log L$. Following the experiment, the results will probably be described by different kinetic functions with different probabilities, which is consistent with some a posteriori distribution $p^*$ (Fig. 2), where $p_m$ is the maximum probability.

Let us compare the amount of information extracted from the thermoanalytical experiment, using traditional and nontraditional methodologies [1] in order to solve the inverse kinetic problem. The nontraditional methodology, based on the principle of complementarity, allows the entire set of kinetic functions to be used for the description of the process, and relies on the distribution shown in Fig. 2. Therefore, the amount of information extracted from the experimental data will be

![Fig. 1 A priori distribution of process description probability by means of  \(x\) kinetic functions](image)

![Fig. 2 A posteriori distribution of process description probability by means of  \(x\) kinetic functions](image)