APPLICATION OF A MOMENTS METHOD AND OF LAPLACE TRANSFORMS TO HEAT TRANSFER EXPERIMENTS*

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(Received June 21, 1979)

We present a mathematical method based on Laplace transform techniques for the analysis of heat capacity and thermal conductivity measurements, for the case of thin film samples on substrates of finite lengths. The method is a further development of the heat pulse technique. This mathematical analysis is capable of separating the heat capacity and thermal conductivity of the sample from those of the substrate, thus eliminating the need for an additional measurement on the substrate alone. This fact substantially reduces the errors and complexity of the experiment and also makes the heat pulse technique the only one capable of obtaining thermal parameters on thin films in a single experiment. The analysis of the experimental data is performed by calculating several moments of the temperature rise in two thermometers as a function of time. Special considerations are taken to adapt the method for on-line computer experiments.

Thin films have very interesting physical properties. However, because of their thickness, only very seldom are the films self-supporting. The use of a substrate, therefore, is the usual solution. This fact does not influence the electrical measurements made on the film, but is a very big drawback in thermal transfer experiments. Even for relatively thick films, the heat capacity and thermal conductivity of the substrate are not negligible compared to those of the film. All the present methods for heat capacity measurements for thin films, including the a.c. method [1], thermal relaxation methods [2], or pulse methods [3–4], lack the possibility of separation of the substrate thermal parameters from those of the film in one experiment. The only way to get absolute results concerning the film alone is to perform an additional experiment on the substrate alone [5] and to subtract the parameters of the substrate from those of the combined assembly of film and substrate.

In this work we present a method based on the heat pulse technique [3–8] by which it would be possible to measure and separate the film and the substrate thermal parameters in one experiment. In the regular heat pulse technique, a \( \delta(t) \)-shaped heat pulse is introduced at one end of a one-dimensional sample. The other end of the sample is attached to a constant-temperature bath. At one point along the sample the time-dependence of the temperature is measured. From the shape of the pulse at that point and the amount of heat introduced to the sample by the \( \delta \) pulse, the heat capacity and thermal conductivity can be deduced with the help of the mathematical method described in references [7–8].

* Supported in part by the Karlsruhe Nuclear Research Center.
In this work the heat transfer equation is solved for a one-dimensional problem with specified boundary conditions. The solution is based on Laplace transforms, and the desired parameters are found by calculating the moments of the temperature rise at two points along the sample. The method is based on the general ideas shown in reference [6], and is therefore one particular case. All the Laplace transforms and their derivatives used in this work do exist mathematically, as proven in reference [6].

The sample covers only one part (usually half) of the substrate, thus dividing the substrate into two non-equivalent regions. One thermometer is placed in each region. By calculating the moments of the temperature rise in each thermometer, we get relations involving the heat capacities and thermal conductivities of the sample and the substrate. Using these relations, the parameters for the sample and substrate alone can be deduced. In Section 1 of this work we present the mathematical problem, its solution, and the calculation of the moments. The experiments performed using this method are especially suitable for on-line computer analysis. In Section 2 we present special considerations necessary for an on-line experiment, especially those needed to overcome problems involving noise and finite measuring time. In order to check the method, we have used the analogy between one-dimensional heat transfer and electrical transmission line equations. We have built an analog electrical circuit, on which we measured successfully the parameters of the circuit by the use of an on-line computer experiment and the mathematical method outlined here. In Section 3 we describe that analog circuit and the experimental results.

Theory

Description of the mathematical problem

A thin elongated sample with the attached heater and two thermometers is shown in Fig. 1. The heater and the thermometers are lines of length W parallel to the width of the sample. Assuming a negligible thickness-to-length ratio for the sample and substrate (usually of the order of 5.10⁻³ or lower), the resulting heat flow geometry is one-dimensional. The direction of this dimension will be called the x-axis. The one-dimensional heat conduction equation for the sample is

\[ K' \frac{\partial^2 T(x, t)}{\partial x^2} = C' \rho \frac{\partial T(x, t)}{\partial t} \]

where \( K' \) is the bulk thermal conductivity, \( C' \) is the bulk specific heat, \( \rho \) is the density and \( T(x, t) \) is the temperature rise at point \( x \) and time \( t \). It is convenient to use one-dimensional quantities \( C = C' \rho Wh \) and \( K = K' Wh \) for the specific heat \( C \) and thermal conductivity \( K \), respectively. We introduce \( R = 1/K \), the thermal resistivity. Actually, \( C \) and \( R \) are the heat capacity and thermal resistivity of unit length of the sample. By using these notations we get the heat transfer equation

\[ J. \text{Thermal Anal. 18, 1980} \]