On R. W. Llewellyn's Rules to Identify Redundant Constraints: 
A Detailed Critique and Some Generalizations

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Eingegangen am 14. April 1978
Revidierte Fassung eingegangen am 29. Januar 1979

Abstract: In his book "Linear Programming" [1964] Llewellyn devoted a chapter to simplifications and reductions of a linear programming problem by means of algebraic rules. These rules are claimed to be rather general. Here we give some counterexamples, where the rules of Llewellyn do not hold. Furthermore we give some general rules to identify redundant constraints in the case Llewellyn considers and show that the original rules of Llewellyn together with an extra condition are a variant of these general rules. Finally we consider the question whether or not the rules of Llewellyn should be used to identify redundant constraints.


1. Introduction

In his book "Linear Programming" published in 1964 Llewellyn devoted a chapter (Chapter 6; esp. 132–138) to simplifications and reductions in linear programming problems by means of algebraic rules. This publication has often been referenced as one of the first attempts to identify redundant constraints in LP problems, or more generally in systems of linear inequalities [see e.g. Thompson/Tonge/Zionts; Gal].

Here we are concerned with the rules Llewellyn gives to identify some special redundant constraints. The rules are intended to identify constraints that are redundant by the presence of one other constraint and the non-negativity constraints on all variables. Basically the same rules are reintroduced in Zeleny [1974].

We briefly review these rules in section 2. Unfortunately, the rules are not as general as is claimed by Llewellyn. Some counterexamples are given in section 3. In section

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4 a general criterion to identify redundant constraints [Gal; Telgen, 1977a] is used to obtain necessary and sufficient conditions for the case Llewellyn considers.

In section 5 it is shown that Llewellyn’s rules together with an extra condition on the signs of the coefficients, are a variant of the general criterion. Finally the question, whether or not the (extended) rules of Llewellyn should be used to identify redundant constraints, is considered both from a practical point of view and in relation with the theory of computational complexity.

2. Llewellyn’s Rules

We consider the system of linear inequalities

\[ \begin{align*}
Ax & \leq b \\
x & \geq 0
\end{align*} \]  

(2.1)

(2.2)

where \( A \) is an \( m \times n \)-matrix; \( x \) and \( 0 \) are \( n \)-vectors and \( b \) is an \( m \)-vector. We denote by \( a_i \) \((i = 1, \ldots, m)\) the \( i \)-th row of \( A \).

Since redundancy is not defined for inconsistent systems, we will assume feasibility of the system (2.1) – (2.2) throughout.

Llewellyn states two rules to identify some redundant constraints from (2.1). A redundant constraint is not defined in his book\(^2\) but we will suppose that Llewellyn implicitly used a widespread definition [see Telgen, 1977b] like:

The \( k \)-th constraint

\[ a_k x \leq b_k \]  

(2.3)

is redundant in the system (2.1) – (2.2) if and only if

\[ a_k x \leq b_k \quad \forall x \in \mathbb{R}^n \quad \bigg| \quad \begin{align*}
a_i x & \leq b_i \\
x & \geq 0.
\end{align*} \]  

(2.4)

The rules, Llewellyn gives, are concerned with the situation in which a constraint in (2.1) is redundant by virtue of one other constraint from (2.1) and all constraints (2.2). In terms of the definition given above, it is the case in which the \( k \)-th constraint is redundant because for some \( s \in (1, \ldots, m) \) \( s \neq k \):

\[ a_k x \leq b_k \quad \forall x \in \mathbb{R}^n \quad \bigg| \quad \begin{align*}
a_s x & \leq b_s \\
x & \geq 0.
\end{align*} \]  

(2.5)

\(^2\) Llewellyn (1964, p. 135): “… the constraint is redundant and can be dropped from the problem without affecting the optimum solution”; this is hardly a definition.