Minimizing the Makespan in a 2-Machine Flowshop with Time Lags and Setup Conditions

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Abstract: A special case of the flowshop problem of sequencing \( n \) jobs on two machines, to minimize the makespan, is solved under setup considerations and with time lags.

Zusammenfassung: In dieser Arbeit wird die Minimierung der Fertigstellungsdauer von \( n \) Aufträgen auf 2 Maschinen behandelt, wobei auch Umrüstzeiten für die Maschinen und Aufbereitungszeiten für die Aufträge in Betracht gezogen werden.

Introduction

Johnson has given a very simple algorithm for the solution of the flowshop problem of scheduling \( n \) jobs on two machines with the assumption that jobs are processed in the order AB of machines A and B, the performance measure being the makespan. In this model it is assumed that as soon as a job finishes processing on A, it goes for processing on B. But there exist problems where a job is still being processed on A while a part of it can go for processing on B. Mitten, Nabeshima and Rinnooy Kan solved problems of scheduling for such a case where time lags are involved. In Rinnooy Kan, problems with arbitrary start lags \( D_i \) and stop lags \( D'_i \) (\( D_i \neq D'_i \)) have been considered.

In the references [Johnson; Mitten; Nabeshima; Rinnooy Kan] setup times are either negligible or are assumed to be included in their processing times. The concept of separated setup times was introduced by Yoshida/Hitomi. There exist problems in big and small industries (like making tins, printing books etc.) which involve separated setup times and time lags. As an illustration in a small industry, consider the manufacturing of Rosin and Turpentine oil from the raw material Resin. To start with the pro-

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cess on the plant, consider the first two parts viz., boiling in a tank and seiving in a
siever. Given 120 tins of resin for the processing, when 60 tins have been boiled in the
tank, they can be sent to the seiver for seiving while the remaining 60 tins are put in
the tank for boiling. In this case, the time lag can be taken as greater than or equal to
the time needed to boil the first batch of 60 tins. Also it happens that before starting
the boiling of the tins in the tank, the fire has to be made up (stroking and adding of
the fuel etc.) and before sending resin from the tins for seiving, the seiver has to be
cleared so that best quality of seived resin is obtained. These requirements lead to-
wards the involvement of separated setup times.

In the present paper we solve the problem of scheduling $n$ jobs on two machines $A$
and $B$ in which time lags and separated setup times are involved. We deal with the
special case when stop lags and start lags are equal, while in general they may be
different. A procedure is evolved to obtain an optimal sequence which minimizes the
makespan under the following constraints: —

Whenever the processing time of a job on machine $A$ is greater (smaller) than its
processing time on machine $B$, the sum of the processing time and setup time of the
job on machine $A$ is also greater (smaller) than the sum of the processing time and
setup time of the job on machine $B$.

These constraints prevail in the example cited above.

Notation

Let

$N$ = set of $n$ jobs $(1, 2, \ldots, n)$.

$A_i$ = Processing time of job $i$ on machine $A$.

$B_i$ = Processing time of job $i$ on machine $B$.

$s_i^A$ = Setup time of job $i$ on machine $A$.

$s_i^B$ = Setup time of job $i$ on machine $B$.

$D_i$ = time lag of job $i$.

$t_i^A$ = starting time of job $i$ on machine $A$.

$t_i^B$ = starting time of job $i$ on machine $B$.

$x_i$ = Idle time on machine $B$ from (the completion of $(i - 1)$-th job on $B$
+ Setup for $i$-th job on $B$) to (the starting of $i$-th job on machine $B$).

Assumptions

The time lags fulfil the following assumptions: —

1. No job $i$ can start processing on machine $B$ until a time-lag $D_i$ has elapsed since $i$
   started processing on machine $A$. 