A MODIFICATION TO THE LINPACK DOWNDATING ALGORITHM

C.-T. PAN

Department of Mathematical Sciences, Northern Illinois University, DeKalb, IL 60115, USA

Abstract.

A LINPACK downdating algorithm is being modified by interleaving its two different phases, the forward solving a triangular system and the backward sweep of Givens rotations, to yield a new forward method for finding the Cholesky decomposition of $R^T R - zz^T$. By showing that the new algorithm saves forty percent purely redundant operations of the original, better stability properties are expected. In addition, various other downdating algorithms are rederived and analyzed under a uniform framework.

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0. Introduction.

The problem of finding the Cholesky factorization of

$$R^T R - zz^T,$$

where $R \in \mathbb{R}^{n \times n}$ is a real upper triangular matrix and $z \in \mathbb{R}^n$ is a column vector (the lower case bold face letters denote the column vectors, $T$ the transpose), is called the Downdating Problem [26]. Throughout this paper it is assumed that $R^T R - zz^T$ is positive definite. The new Cholesky factor (compared to $R$), or the downdated Cholesky factor $D$, then exists and satisfies

$$D^T D = R^T R - zz^T,$$

where the upper triangular matrix $D$ is unique up to the signs of the rows of $D$. Without loss of generality, we assume that both $R$ and $D$ have positive diagonal entries.

There are numerous applications of downdating the Cholesky and the QR factorization. For instance, in the optimization field, the field that motivates the
problem, the downdating algorithm is applied to such as the linear programming problem [13,25] and the rank-one modification in quasi-Newton methods for unconstrained optimization problems [14]. In the digital signal processing area, the downdating algorithm is part of the windowed recursive least squares filtering algorithm [1]. It is especially worth noting that in several recently proposed fast QR factorization algorithms for Toeplitz matrices (for example, [4,8,27]), the downdating problem plays a key role. The complexity and stability of their algorithms depend crucially on the complexity and stability of the downdating algorithms discussed here. More recently, the downdating algorithms have been applied to modify the inverse of the Cholesky factorization, in connection with solving the bottle-neck problem (to avoid a triangular system solver) in the parallel implementation of the recursive least squares algorithm used in signal processing [22].

The counter-problem of downdating is better known as the updating problem, i.e., to find an upper triangular matrix $U$ such that

$$U^T U = R^T R + zz^T,$$

where $R$ and $z$ are the same as given in (1). In contrast to the downdating problem, the updating problem always has a solution since $R^T R + zz^T$ is guaranteed to be positive definite. Moreover, the straightforward updating algorithm using Givens rotations is undoubtedly stable according to the classical work of Wilkinson [28].

For the downdating problem, the algorithm itself is not so straightforward, and the stability is always in question, principally because of the subtraction in formula (1). It is for these reasons that the downdating problem has captured so much attention in the field of numerical analysis in the past two decades (see [18] and [2]).

The original downdating algorithm, which is now called the hyperbolic rotation algorithm, was first proposed by Golub [15]; later Saunders [25] proposed a significantly different method, which is now written as a subroutine in LINPACK. Stewart [26] shows that Saunder's algorithm is stable in the sense that the computed Cholesky factor is very near the factor obtained by downdating with a slightly perturbed vector $z$. However, he cautions that the downdating problem can be very ill-conditioned and then $D$ can be inaccurate. Golub's algorithm has long been suspected to be unstable, but a recent paper of Alexander, Pan, and Plemmons [1] shows that it gives a forward error of the same magnitude as the LINPACK algorithm. Our numerical tests [20] confirm that the two algorithms have nearly equal computed answers in $D$, with the LINPACK algorithm being almost always marginally more accurate than the hyperbolic one. In terms of operation counts, the hyperbolic algorithm requires $2n^2$ multiplications while the LINPACK one requires $\frac{5}{2}n^2$ multiplications.

There are several variations of the hyperbolic rotation downdating algorithm. The one proposed by Chambers [7] (see also [18]) which requires $2n^2$ multiplications was recently analyzed by Bojanczycyk, Brent. Van Dooren and de Hoog [3] and shown to possess the same kind of stability as the LINPACK algorithm. Analogous to the fast Givens rotations [11], one can modify the hyperbolic rotations in a like