Part II

NUMERICAL MATHEMATICS
THE MÜHLBACH–NEVILLE–AITKEN ALGORITHM
AND SOME EXTENSIONS

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Abstract.
A new proof of the Mühlbach–Neville–Aitken algorithm for interpolation by a linear family of functions forming a Chebyshev system is given. This proof is based on Sylvester's identity for determinants. The algorithm is then applied to the general interpolation problem, and applications to orthogonal polynomials and Padé-type approximants are treated. Finally the extension to rational interpolation is also studied.

I. Introduction.
A general extrapolation algorithm including almost all the convergence acceleration methods actually known has been recently discovered by Hävie [8] and Brezinski [3]. The question immediately arises if a general interpolation algorithm (i.e. an algorithm for interpolation by a linear family of functions forming a Chebyshev system which is similar to the Neville–Aitken scheme) exists or not. In fact such an algorithm has been obtained by Mühlbach [13, 14] some years ago.

The aim of this paper is twofold. First I shall present a new proof of the Mühlbach–Neville–Aitken algorithm (called the MNA-algorithm) similar to the proof I gave for the general extrapolation algorithm. This proof, which is shorter and, I think, simpler than the successive proofs given by Mühlbach, is based on Sylvester's identity for determinants. Mühlbach's notations have been slightly changed to easier ones. The second aim of this paper is to show that the MNA-algorithm can be used for the general interpolation problem as described, for example, by Davis [6]. Applications to orthogonal polynomials and Padé-type approximants are given. Finally the case of rational interpolation is studied.

II. The Mühlbach–Neville–Aitken algorithm.
Let \((f_i)_{i \geq 0}\) be a family of functions \(f_i : G \to K\) where \(G\) is an arbitrary set and \(K\) a commutative field of characteristic zero. The functions \(f_i\) will be assumed to form a Chebyshev system which means that all the Gram determinants \(|f_i(x_j)|_{i,j=0,\ldots,k}\) are different from zero whenever \(x_0, \ldots, x_k\) are distinct points in \(G\) (for an extensive study of Chebyshev systems see the papers by Mühlbach [13, 14] and the book by Karlin and Studden [9]).

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