ON NUMERICAL EVALUATION OF DOUBLE INTEGRALS OF AN ANALYTIC FUNCTION OF TWO COMPLEX VARIABLES

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Abstract.

A seventh degree rule of the non-product type has been constructed for numerical evaluation of double integrals of an analytic function of two complex variables by choosing a set of 17 points from the set of 25 points needed in the product Birkhoff-Young rule of fifth degree. An asymptotic error estimate for this rule has been determined and the rule has been numerically tested.

1. Introduction.

Birkhoff and Young [3], Lether [7], and Tošić [9] have considered the problem of numerical evaluation of complex definite integrals of an analytic function of one complex variable. In our opinion, the problem of numerical approximation of complex multiple integrals has not received sufficient attention unlike the problem of numerical approximation of real multiple integrals for which exhaustive references have been given in Haber [6], Stroud [8], and Engels [5].

Recently Acharya and Das [1] and Das, Padhy and Acharya [4] have investigated the 25-point fifth degree product rule based on the Birkhoff-Young 5-point fifth degree rule for approximation of the complex double integral $I(f)$

$$I(f) = \int_{L_1} \int_{L_2} f(z^{(1)}, z^{(2)}) dz^{(1)} dz^{(2)},$$

where $L_j$ is a directed line segment from the point $z^{(j)}_0 - h_j$ to $z^{(j)}_0 + h_j$ in the $z^{(j)}$-plane $(j = 1, 2)$ and $f$ is analytic in the domain $\Omega_1 \times \Omega_2$, where

$$\Omega_j = \{z^{(j)} : |z^{(j)} - z^{(j)}_0| \leq R_j, \quad R_j > |h_j|\} \quad (j = 1, 2).$$

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Later for approximation of $I(f)$, Acharya and Das [2] have derived the 17-point fifth degree rule of the non-product type by discarding certain points from the set of 25 points meant for the product layout based on the 5-point Birkhoff and Young rule [3].

Although problem (1.1) can be reduced to the approximation of two real double integrals over a quadratic region by introduction of two new variables $z^{(j)} = z_0^{(j)} + x_j h_j$, $x_j \in [-1, 1]$ ($j = 1, 2$), we will solve it directly and simply using complex arithmetic by deriving a quadrature rule of seventh degree using only 17 points.

2. Formulation of the fifth degree rules.

Let $k$ be a non-zero real parameter and $z_m^{(j)} = z_0^{(j)} + k i^m h_j$, where $i = \sqrt{-1}$, $j = 1, 2$ and $m = 1(1)4$. Then $z_s^{(j)}$ and $z_l^{(j)}$ are the end points of the paths of integration $L_j$ ($j = 1, 2$). Let us introduce the following sets of points:

$$P_j = \{z_0^{(1)}, z_1^{(1)}, z_2^{(1)}\},$$

$$Q_j = \{z_0^{(2)}, z_1^{(2)}, z_2^{(2)}\},$$

$$S_j = \{z_1^{(1)}, z_2^{(1)}\},$$

$$T_j = \{z_1^{(2)}, z_2^{(2)}\},$$

$$Y_j = \{z_0^{(1)}\}$$

$j = 1, 2$.

We consider the following sets each of cardinality 13:

$$A = (P_1 \times P_2) \cup (Y_1 \times T_2) \cup (T_1 \times Y_2),$$

$$B = (Q_1 \times Q_2) \cup (Y_1 \times S_2) \cup (S_1 \times Y_2),$$

where $\times$ denotes Cartesian product. Finally $A \subseteq \Omega_1 \times \Omega_2$ and $B \subseteq \Omega_1 \times \Omega_2$ if $k \in (0, 1]$. Let $f(s_p^{(1)}, z_q^{(2)}) = f_{pq}$ ($p, q = 0(1)4$).

For constructing the 13-point rules of degree five, we take $A$ and then $B$ as the sets of interpolating points. With $A$ as the set of interpolating points the following 13-point rule for approximation of $I(f)$ is proposed:

$$I(f) \cong Q_1(f) = h_1 h_2 [a_0 f_{00} + a_1 (f_{10} + f_{01} + f_{03} + f_{30})$$

$$+ a_2 (f_{20} + f_{02} + f_{04} + f_{40}) + a_3 (f_{11} + f_{13} + f_{31} + f_{33})].$$

(2.1)

Similarly with $B$ as the set of interpolating points the following 13-point rule for the numerical approximation of $I(f)$ is proposed:

$$I(f) \cong Q_2(f) = h_1 h_2 [b_0 f_{00} + b_1 (f_{10} + f_{01} + f_{03} + f_{30})$$

$$+ b_2 (f_{20} + f_{02} + f_{04} + f_{40}) + b_3 (f_{22} + f_{24} + f_{42} + f_{44})].$$

(2.2)