WEIGHT DIAGRAMS FOR LIE GROUP REPRESENTATIONS:
A COMPUTER IMPLEMENTATION OF FREUDENTHAL'S ALGORITHM IN ALGOL AND FORTRAN
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Abstract.
Implementations in FORTRAN and ALGOL of the Dynkin and Freudenthal algorithms for computing weight systems and for determining the multiplicities of the weights for irreducible representations of simple Lie algebras are described. Reasonable computing times are found for algebras of rank less than or equal to 8 and for representations of dimension less than 1000.

I. Introduction.
It is the purpose of this paper to provide an account of some experience that we have gained over the past year with machine computations of characters of irreducible representations of simple Lie algebras. These computations were for the most part carried out in the summer of 1968 on the UNIVAC 1108 computer at the Carnegie-Mellon University, using UNIVAC 1107/8 ALGOL and FORTRAN V implementations of the Dynkin and Freudenthal algorithms. An ALGOL 60 adaptation of these programs is given in the appendix to this paper. We have found from the actual runs that these algorithms provide an entirely practical procedure for computing weight diagrams of Lie modules. In a survey of weight diagrams of irreducible modules of dimension up to 1000 for simple Lie algebras of rank up to 8, it was found that the FORTRAN version takes on the average only 2.5 seconds to compute a single weight diagram including multiplicities. The method is practical therefore for the special unitary groups up to $SU(9)$, for the orthogonal groups up to $SO(17)$, the symplectic groups up to $Sp(8)$, as well as for all five exceptional simple Lie groups $G_2$, $F_4$, $E_6$, $E_7$, and $E_8$, the last three of these having comparatively few representations of dimension less than a thousand however. These limits on rank and dimension are not rigid.
limits, but were imposed to keep the total computation time for the sur-
vey within 20 minutes.

There are essentially three methods available for computing charac-
ters of Lie modules which are not specialized to any particular type of
simple Lie algebra. One of these methods dates back [35, 36, 37, 38] to
the early researches of Weyl in 1925–26. The Weyl method of girdle
division has been used for hand computations of characters of rank two
simple Lie algebras by Behrends, Dreitlein, Fronsdal and Lee [4]. A
second method, due to Freudenthal, makes use of a simple algorithm
[13, 14, 15] for computing characters. This method has the advantage
that it avoids the use of the group generated by Weyl reflections and is
therefore suitable for programming on a computing machine, as discussed
in the sequel. A third method due to Kostant makes use of both the
Weyl group and partition function methods [21, 22, 34]. Kostant's
method as well as Weyl's original method are sometimes considered to
be more elegant than Freudenthal's method from a theoretical point of
view in that these methods make use of closed formulas rather than
algorithmic procedures.

A number of laborious hand computations of characters were pub-
lished recently in connection with recent speculations [27, 30] about
hadron symmetry schemes in elementary particle physics. The most
complete calculations of this sort seem to have been carried out by
Konuma, Shima and Wada [20]. The difficulty of the hand computations
led to considerable discussion of improvements in computation methods.
It was suggested by Antoine and Speiser [2, 3] that the Kostant formula
might save computation time, particularly for high rank Lie algebras
and for high dimensional representations. Klimyk [40] also suggested
another method which is a modification of the Kostant procedure. The
discouragingly few results reported in all these papers supports the view
that all known methods are impractical for hand computations of weight
diagrams beyond rank three. For hadron symmetry schemes based on
the rank five Lie algebra of the group $SU(6)$ it appeared to be imprac-
tical to use weight diagrams at all in practical calculations. Once the
computer is brought into the picture however, the situation improves
dramatically. Our experience shows that even the simple Freudenthal
algorithm is quite adequate for all applications in physics foreseeable in
the near future. In particular, all of the existing published weight dia-
gram computations can be reproduced in a matter of seconds by the
computer and serve as a check that there are no errors in the programs.
High rank groups proposed in physics such as $SO(8)$ and $SU(6)$ pose no
problem. It is conceivable to us that there may be applications outside