Part II

NUMERICAL MATHEMATICS
PHASE-LAG ANALYSIS OF EXPLICIT NYSTRÖM METHODS FOR $y'' = f(x, y)$

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Abstract.

We examine phase-lag (frequency distortion) of the two-parameter family $M_4(\alpha, \beta)$ of fourth order explicit Nyström methods of [1] by applying these to the test equation: $y'' + \lambda^2 y = 0$, $\lambda > 0$. While the method $M_4(1/6, 5/6)$ possessing the largest interval of periodicity of size 3.46 has a phase-lag of $(1/4320)H^4$ ($H = \iota h$, $h$ is the step-size), we show that there exist two fourth order methods of $M_4(\alpha, \beta)$ for which the phase-lag is minimal and of size $(1/40320)H^6$; interestingly, both methods also possess a sizable interval of periodicity of length 2.75 each.

1. For the special second order initial value problem:

(1) $y'' = f(x, y), \ y(x_0) = y_0, \ y'(x_0) = y'_0,$

consider an explicit Nyström method of order $s$, based on $s - 1$ function evaluations, defined by

$$
\begin{align*}
y_{k+1} &= y_k + hy'_k + h^2 \sum_{j=1}^{s-1} a_j K_j + t_k(h), \\
y'_{k+1} &= y'_k + h \sum_{j=1}^{s-1} b_j K_j + t'_k(h),
\end{align*}
$$

where

$$
K_i = f(x_k + \alpha_i h, y_k + \alpha_i y'_k + h^2 \sum_{j=1}^{i-1} \beta_{ij} K_j), \quad i = 1(1)s - 1,
$$

and $t_k(h), t'_k(h) = O(h^{s+1})$. Chawla and Sharma [1] discussed periodicity of these methods of orders $s = 2, 3$ and 4. It was shown in [1] that there exists a uniquely determined second order method $M_2$ possessing an interval

Received December 1984. Revised August 1985.