COMPUTER SIMULATION OF TRANSVERSE STRING VIBRATIONS

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Abstract.

A new approach to the study of linear and nonlinear string vibrations is developed by means of discrete simulation. Computer examples using different laws of tension are described and compared. The method is exceptionally fast and allows for a wide range of parameter choices.

1. Introduction.

Elastic string vibration has been a subject of study by mathematicians, physicists and engineers for many years (see, e.g., references [1], [2], [4]-[6], and the additional references contained therein). For such problems, we will develop in this paper a discrete approach which will enable us to simulate easily on a digital computer both linear and nonlinear behavior for a large class of problems.

2. Discrete Strings.

A discrete string is one composed of a finite number of particles. It will be treated as an ordered set of \( n + 2 \) circular, homogeneous particles \( C_i, i = 0, 1, 2, \ldots, n, n + 1 \), each of mass \( m \), with respective centers \( (x_i, y_i) \). Our problem will be that of describing the return of a discrete string to a position of equilibrium after release from an arbitrary position of tension, when \( n \) is relatively small. (In this paper \( n = 499 \). Excellent computer runs were made for \( n = 999 \), but funds were not available to ensure the stability of the calculations.) The resulting motion can be considered as an approximation to that of a real string.

We proceed under the assumptions that \( C_0 \) and \( C_{n+1} \) are fixed with \( x_0 = y_0 = y_{n+1} = 0 \), while \( C_1, C_2, \ldots, C_n \) are free to move, and that each particle can move in a vertical direction only. Let \( x_0 < x_1 < x_2 < \ldots < x_n < x_{n+1} \) and \( x_i - x_{i-1} = \Delta x, i = 1, 2, \ldots, n+1 \). At time \( t_k, k = 0, 1, 2, \ldots \), measured in seconds, denote the centers of \( C_{i-1}, C_i, C_{i+1}, i = 1, 2, \ldots, n \), by \( (x_{i-1}, y_{i-1, k}), (x_i, y_{i, k}) (x_{i+1}, y_{i+1, k}) \), respectively, where each coordinate

Received June 5, 1971.
is measured in feet. In studying the motion of each $C_i$, we consider only tensile and gravitational forces, and for this purpose let $T_{i-1,i}$ be the tensile force between $C_{i-1}$ and $C_i$ and $T_{i,i+1}$ be the tensile force between $C_i$ and $C_{i+1}$. Then, to each particle, Newton's dynamical equation will be applied in the form [4]

\[
ma_{i,k} = |T_{i,i+1}| \frac{y_{i+1,k} - y_{i,k}}{[(Ax)^2 + (y_{i+1,k} - y_{i,k})^2]^{\frac{1}{2}}} \\
- |T_{i-1,i}| \frac{y_{i,k} - y_{i-1,k}}{[(Ax)^2 + (y_{i,k} - y_{i-1,k})^2]^{\frac{1}{2}}} - mg; \quad i = 1, 2, \ldots, n.
\]

The initial-value problem to be studied is that of fixing $(x_i, y_{i,0})$, $i = 1, 2, \ldots, n$ and of describing then the resulting motion determined by (2.1) when the string is released from its initial position, at which it is considered to be at rest. If the velocity of $C_i$ at $t_k$ is denoted by $v_{i,k}$.

Fig. 1.