Part I

EDUCATIONAL PAPERS
ON BINARY OPERATORS AND THEIR DERIVED RELATIONS

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Dedicated to Peter Naur on the occasion of his 60th birthday

This note contains about half a dozen charming little theorems, none of which I expect to be new. It has been written because, though their proofs are very simple, the theorems are not half as well-known as I feel they deserve to be. I hope the reader will be as pleasantly surprised by the independence of the links between pairs of well-known concepts as I was when I composed this collection.

A few notational remarks first.

(i) I shall use ( , == , ~ , => , '), here listed in the order of decreasing binding power.

(ii) In the proofs, the hint "Leibniz" refers to the fact that we may "substitute equals for equals", i.e.

\[ x = y \Rightarrow f(x) = f(y) \]

or, equivalently,

\[ x = y \land b(x) \Rightarrow b(y). \]

(iii) In hints, the symbol := is used to denote the instantiating substitution.

\[ * \quad * \quad * \]

In terms of some infix operator • we define the relation \( \triangleleft \) by

\[ (\forall x, y :: x \triangleleft y \equiv x \cdot y = x). \]

(Example: if we choose "greatest common divisor" for •, \( \triangleleft \) becomes "divides").

Received May 1987.