Part II

NUMERICAL MATHEMATICS
ON THE NUMERICAL SOLUTION OF AN
INTEGRO-DIFFERENTIAL EQUATION ARISING
FROM WAVE-POWER HYDRAULICS

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Abstract.

A convergence analysis is presented for an implicit linear multistep method for solving
integro-differential equations associated with multivalued maximal monotone mappings
with special reference to an equation describing the wave induced motion of a body
damped by Coulomb friction.

1. Introduction.

Our purpose is to present the derivation and analysis of an implicit
discretization method for solving the general integro-differential equation

\[ dw(t)/dt + M(w(t)) + N(t; w(t)) = F(t, w(t)); w(0) = w_0 \]

where \( w(t) \in H, H \) is the Hilbert space \( \mathbb{R}^k \) with the Euclidean inner product,
\( N(t; w(t)) \) is a linear functional, \( F(t, w(t)) \) is a Lipschitz mapping from \([0, T] \times H\)
to \( H \) and \( M(w(t)) \) is a multivalued maximal monotone mapping from \( H \) to itself,
i.e.

\[ (M(x) - M(y), x - y) \geq 0, \forall x, y \in H \]

\[ (b) \text{ for each } b \in H \text{ and } \gamma > 0 \text{ there exists a unique } x \text{ such that } x + \gamma M(x) = b. \]

The study is motivated by the work of Count ([2], [3]) for the Central
Electricity Generating Board on the feasibility of wave power. One idea is to use a
hydraulically linked "duck" to obtain energy from small amplitude waves. The
hydraulic linkage essentially acts as a friction force which extracts energy from the
system and modelling this by Coulomb friction, Count obtained the scalar second
order integro-differential equation

\[ \ddot{x} + \mu \text{sign} \dot{x} + \int_0^t k(t, s)x(s)ds = f(t, x(t)) \]

where \( x(t) \) denotes the inclination of the duck at time \( t \) and \( \mu \text{sign}\dot{x} (\mu > 0) \) is the
Coulomb or dry frictional force defined by

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