ON SEMI-ANCHORS OF A TRANSLATION PLANE

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In this note examples of semi-anchors are given.

1. Introduction. Recently F. Radó has introduced the concept of "semi-anchor", which turned out to be useful in studying extensions of collineations defined on subsets of a plane [1], [2].

Let $\Pi, \Pi'$ be translation planes, $\mathcal{P}, \mathcal{P}'$ their sets of points, $\mathcal{L}, \mathcal{L}'$ their improper lines and $T, T'$ the corresponding translation groups. Given a set $\mathcal{A} \subset \mathcal{P}$, the injective map $\varphi : \mathcal{A} \rightarrow \mathcal{P}$ is called a collineation, if it maps proper (improper) points in proper (improper) points and three points in $\mathcal{A}$ are collinear if and only if their images are collinear. The set of these maps is denoted by $\varphi(\mathcal{A}, \mathcal{P}')$. We shall use the following notations: $\mathcal{A}_o = \mathcal{A} \setminus L_{\infty}$, $\mathcal{A}_\tau = \mathcal{A}_o \cup \tau(\mathcal{A}_o)$, where $\tau \in T$, $\tau(\mathcal{A}) = \{ \tau | \tau \in T, \mathcal{A}_\tau \neq \emptyset \}$. A set $\mathcal{A} \in \mathcal{P}$ is said to be compatible with the translation $\tau \in \tau(\mathcal{A})$, if for every second translation plane $\Pi'$ and any $\varphi \in \varphi(\mathcal{A}, \mathcal{P}')$ there exists $\tau' \in T'$ such that

$$\forall M \in \mathcal{A}_\tau, \tau' \varphi(M) = \tau \tau(\mathcal{M}).$$

A set $\mathcal{A}$ is called a semi-anchor if it is compatible with every $\tau \in \tau(\mathcal{A})$.

Our aim is to indicate more general classes of semi-anchors than the examples in [1].

2. Preliminaries. Let $\tau_o \in T \setminus \{1\}$ be a given translation. We say that the set $\mathcal{A}_o$ belongs to $\tau_o$, when $\mathcal{A}_o = \mathcal{A}_o \cup \mathcal{A}_{\tau_o}$ (i.e. $M \in \mathcal{A}_o$ implies $\tau_o(M) \in \mathcal{A}_o$ or $\tau_o^{-1}(M) \in \mathcal{A}_o$).

Let $P_0, P_1, P_2, P_3$ be four points in $\mathcal{P}$ and suppose that there exists a translation $\tau \in T$ in such a way that $\tau(P_0) = P_1$, $\tau(P_3) = P_2$. It follows that there is a $\tau' \in T$ such that $\tau'(P_0) = P_3$.
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= \mathcal{P}_3, \tau'(P_1) = P_2. The set \{P_0, P_1, P_2, P_3\} is called a parallelogram if \tau, \tau' \neq 1. The pairs (P_0, P_1), (P_1, P_2), (P_2, P_3), (P_3, P_1) are called adjoining vertices, while the pairs (P_0, P_2), (P_1, P_3) are called opposite vertices. We shall say that \( I_1 = \text{cent} = P_0 P_1 \cap l_\infty \) and \( I_2 = \text{cent} = P_0 P_3 \cap l_\infty \) are the improper diagonal points of the parallelogram \( P_0 P_1 P_2 P_3 \).

The parallelogram \( P_0 P_1 P_2 P_3 \) is said to be non-singular, if the points \( P_0, P_1, P_2, P_3 \) do not lie on a line (\text{cent} \neq \text{cent}''), it is singular in the other case (\text{cent} = \text{cent}''). A singular parallelogram \( P_0 P_1 P_2 P_3 \) contained in \( \mathcal{A}_\tau \) is said to be attached to \( \mathcal{A}_\tau \), if there exist \( M, N \in \mathcal{A}_\tau \setminus P_0 P_1 \) and \( \tau, \tau' \in \mathbb{T} \) such that \( \tau(P_0) = P_1 \) or \( \tau(M) = N \).

We shall distinguish four types of parallelograms contained in \( \mathcal{A}_\tau = \mathcal{A}_{\tau_0} \cup \mathcal{A}_{\tau_0}'' \).

Type A: \( P_i \in \mathcal{A}_{\tau_0} \), \( i = 0, 1, 2, 3 \) or \( P_i \in \mathcal{A}_{\tau_0}'', i = 0, 1, 2, 3 \);

Type B: three vertices are in \( \mathcal{A}_{\tau_0} \), one in \( \mathcal{A}_{\tau_0}'' \) or vice-versa.

Type C: two adjoining vertices are in \( \mathcal{A}_{\tau_0} \), the other two in \( \mathcal{A}_{\tau_0}'' \).

Type D: two opposite vertices are in \( \mathcal{A}_{\tau_0} \) and two opposite vertices in \( \mathcal{A}_{\tau_0}'' \).

The parallelograms \( P_0 P_1 P_2 P_3 \) and \( Q_0 Q_1 Q_2 Q_3 \) of type A are called associated if \( Q_i = \tau_i(P_i) \) or \( P_i = \tau_i(Q_i) \) for \( i = 0, 1, 2, 3 \). A parallelogram is called special if one improper diagonal point coincides with \text{cent} \( \tau_0 \).

A line of the plane \( \Pi \) is called associated to \( \mathcal{A}_\tau \) when it contains at least two points of \( \mathcal{A}_\tau \). We denote by \( a_i(\mathcal{A}) \) the set of those improper points which are incident with at least \( i \) distinct lines associated to \( \mathcal{A}_\tau \).

3. Two classes of semi-anchors. Let the characteristic of