On the "Bayesification" of the Minimax Principle

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Summary: The term "Bayesification" is introduced as meaning the adoption of criteria, that keep the Bayes type of solution as far as the knowledge of the probability distribution over the states of nature extends. A general decision model in Wald’s sense is developed, Bayes and minimax solution are defined and reasons are given for the Bayesification. Neither the Bayes nor the minimax criterion per se allow Bayesification. In order to get a Bayesification in the above sense the Bayes type of solution is extended to a certain combination of Bayes and minimax type of solution. This extension is generalized in order to provide a mechanism that allows the decision maker to incorporate into the decision criterion any a priori information available. It is shown that special cases of the generalized criterion are Bayes solution, minimax solution, and a solution analogous to that of HODGES and LEHMANN. Another special case of the generalized criterion is shown as a special case of a criterion proposed by SCHNEEWEISS.

1. Introduction

1.1 In a recent paper [9] I tried to provide the minimax risk criterion with a general structure adaptable to various generic situation types. The specializations made in that paper were mainly devoted to the rôle time plays in decision problems.
In the present paper I want to extend the Bayes as well as the minimax type of solution in such a way as to approach stepwise that general structure of the minimax principle. At the same time I try to clarify some ideas of the original paper concerning various degrees of uncertainty.

I call this attempt "Bayesification" of the minimax principle because of the following leading idea which—in order to avoid possible misunderstandings—I shall define forthwith.

By "Bayesification" I understand a certain behaviour of the decision maker concerning the adoption of decision criteria. The decision maker has to make a choice between the application of the so-called Bayes type of solution on the one hand and the application of the minimax type of solution on the other.

I assume the decision maker principally wants to adopt the Bayes type as an immediate expression of objective rationality.

But this desire in most practical cases cannot be fulfilled because of lack of knowledge of the a priori distribution over the states of nature. He therefore is forced to adopt the minimax type. I consider it as irrational if the decision maker then switches to the opposite extreme, i.e. to the minimax type in its pure form.

Rather the decision maker is recommended to keep the Bayes type as far as his knowledge of the probability distribution over the states of nature extends. The decision rationale he follows then is to eliminate as many states of nature as possible (i.e. justified by the amount of information he possesses) from the domain of the minimax criterion and to subject them to the Bayes criterion. This rationale I call "Bayesification of the minimax principle of behaviour".

1.2 The decision maker, of course, is an idealized figure who on the one hand possesses unlimited logical capacities and unlimited computing facilities, who also finds no displeasure in thinking, but who, on the other hand, is confronted with a conflict.

A certain number of actions $a$ are at his disposal, and he faces a certain number of states of nature $F$. If he knows which $F$ is the true one, then he has to make a decision under certainty. This case we omit from our considerations. If he does not know which $F$ is the true one then he is confronted with a decision under uncertainty. The uncertainty can appear in three different forms:

(1) Nature is his opponent and wants to defeat him. This mythological case I want to exclude.

(2) Nature can appear as an opponent which itself is not rational but at the same time not measurable. This is the case of complete uncertainty.

(3) Nature can appear as an opponent that is not rational but measurable, i.e. the decision maker knows the true probability distribution $\lambda$ over the states of nature. This is the case of risk.