Boehmians were first introduced by a purely algebraic construction presented in [5]. Applications of the method to function spaces yield various spaces of generalized functions (see [6], [7] and [8]). In those examples, the construction is based on the concept of convolution. The fact, that the convolution of a continuous function on \( R^q \) with a function with compact support in \( R^q \) is always a continuous function on \( R^q \), is of basic importance for the construction. The situation complicates in the case of functions defined on an open subset of \( R^q \): the convolution of such a function with a function with compact support is not usually defined for all points from that open set. Therefore the algebraic method cannot be used.

It is proved in [6], that the space of Boehmians on \( R^q \) can be equivalently defined as the completion of the space of continuous functions on \( R^q \) with respect to a special type of convergence, called \( A \)-convergence. The definition of Boehmians on open sets, presented in this paper, is based on that fact.

1. Let \( \Omega \) be a fixed open set in \( R^q \). In this paper we are going to use the following notation:

- \( C(\Omega) \) the space of all continuous complex-valued functions on \( \Omega \),
- \( C_C \) the space of all continuous real-valued functions on \( R^q \) with compact support,
- \( |x| = (x_1^2 + \ldots + x_q^2)^{1/2} \) for \( x = (x_1, \ldots, x_q) \in R^q \),
- \( B_{\varepsilon} = \{x \in R^q: |x| < \varepsilon\} \) for \( \varepsilon > 0 \),
- \( A_{\varepsilon} = \{x \in R^q: x + B_{\varepsilon} \subset A\} \) for \( \varepsilon > 0 \) and \( A \subset R^q \),
- \( s(\varphi) = \inf \{\varepsilon > 0: \text{supp } \varphi \subset B_{\varepsilon}\} \) for \( \varphi \in C_C \),
- \( \overline{A} \) the closure of \( A \) for \( A \subset R^q \).

The concept of convolution plays the crucial role in the construction of Boehmians. Let \( f \in C(\Omega) \) and \( \varphi \in C_C \). By the convolution \( f \ast \varphi \) we mean the function

\[
(f \ast \varphi)(x) = \int_{\text{supp } \varphi} f(x-u)\varphi(u) \, du
\]

which is well defined for all \( x \in R^q \) such that \( x - \text{supp } \varphi \subset \Omega \). Clearly, it may happen that there is no such \( x \).
Let \( f \in C(\Omega) \) and let \( U \) be an open set such that for some \( \varepsilon > 0 \) we have \( \overline{U + B_\varepsilon} \subseteq \Omega \). Then there exists \( g \in C(\mathbb{R}^d) \) such that \( f = g \) on \( U + B_\varepsilon \). Therefore
\[
(f * \varphi)(x) = (g * \varphi)(x)
\]
for every \( x \in U \) and every \( \varphi \) such that \( s(\varphi) < \varepsilon \). Consequently, the defined convolution has all basic properties of the convolution defined globally (over entire \( \mathbb{R}^d \)). For example, \( ((f * \varphi) * \psi)(x) = (f * (\varphi * \psi))(x) \) for every \( x \in \Omega \) where both sides are defined.

Denote by \( S_0 \) the subset of \( C_c \) consisting of all those non-negative functions \( \varphi \) such that \( \int \varphi = 1 \). A sequence \( \delta_n \in S_0 \) is called a delta sequence, if \( s(\delta_n) \to 0 \) as \( n \to \infty \). It is important to note that, if \( \varphi_n \) and \( \psi_n \) are delta sequences, so is the sequence of convolutions \( \varphi_n * \psi_n \).

Let \( K \) be a compact subset of \( \Omega \) and let \( f_n \in C(\Omega) \) for \( n = 1, 2, \ldots \). If \( \delta_n \) is a delta sequence, then the sequence of convolutions \( f_n * \delta_n \) is defined on \( K \) for all sufficiently large \( n \in \mathbb{N} \), say all \( n \) greater than some \( n_0 \). If, moreover, the sequence \( f_n * \varphi_n \) (where \( n_0 + 1, n_0 + 2, \ldots \) converges uniformly on \( K \), we simply say that \( f_n * \varphi_n \) converges uniformly on \( K \).

It is known and easy to prove that if \( f \in C(\Omega) \) and \( \delta_n \) is a delta sequence, then the sequence of convolutions \( f * \delta_n \) converges to \( f \) uniformly on every compact subset of \( \Omega \).

**Definition 1.1.** A sequence of functions \( f_n \in C(\Omega) \) is called \( \Delta \)-convergent to \( f \in C(\Omega) \), if for each compact \( K \subseteq \Omega \) there exists a delta sequence \( \delta_n \) such that the sequence of convolutions \( (f_n - f) * \delta_n \) converges to zero uniformly on \( K \). In this case we write \( f_n \xrightarrow{\Delta} f \).

**Remark.** \( \Delta \)-convergence is weaker than the usual convergence in \( C(\Omega) \); if a sequence \( f_n \in C(\Omega) \) converges to \( f \in C(\Omega) \) uniformly on each compact subset of \( \Omega \), then the sequence of convolutions \( (f_n - f) * \delta_n \), where \( \delta_n \) is any delta sequence, converges to zero uniformly on every compact subset of \( \Omega \). This property follows immediately from the following

**Lemma 1.2.** Let \( K \subseteq \mathbb{R}^d \) be a compact set. Let \( K' = K + B_\varepsilon \) for some \( \varepsilon > 0 \). For any function \( f \) continuous on \( K' \) and any \( \varphi \in C_c(\mathbb{R}^d) \) such that \( s(\varphi) < \varepsilon \) we have
\[
\max_{x \in K'} |(f * \varphi)(x)| \leq \max_{x \in K'} |f(x)| \cdot \int_{\mathbb{R}^d} |\varphi(x)| \, dx.
\]

**Proof.** Let \( x \in K \). Then
\[
|(f * \varphi)(x)| \leq \int_{\text{supp} \varphi} |f(x - u)| |\varphi(u)| \, du \leq \max_{x \in K'} |f(x)| \cdot \int_{\mathbb{R}^d} |\varphi(x)| \, dx.
\]
Since the inequalities hold for every \( x \in K \), the desired inequality follows.

**Lemma 1.3.** If \( f_n \xrightarrow{\Delta} f \), \( g_n \xrightarrow{\Delta} g \), \( f_n, g_n, f, g \in C(\Omega) \), \( \alpha_n \rightarrow \alpha \) (\( \alpha_n, \alpha \) are complex scalars), then \( (f_n + g_n) \xrightarrow{\Delta} f + g \) and \( \alpha_n f_n \xrightarrow{\Delta} \alpha f \).

**Proof.** Let \( K \) be a compact subset of \( \Omega \) and, let \( \varphi_n \) and \( \psi_n \) be delta sequences such that both sequences \( (f_n - f) * \varphi_n \) and \( (g_n - g) * \psi_n \) converge to zero uniformly...