COROLLARY 2 [7]. \( \text{Th}(L_e) \neq \text{Th}(L_r) \) for \( r \in \{d, \rho, \iota\} \).

LITERATURE CITED


COVERINGS IN THE LATTICE OF \( \ell \) -VARIETIES

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An \( \ell \) -variety \( V \) in which the identity \( (x \land y \land z) \lor e = e \) is valid is said to be a 0-approximable \( \ell \) -variety. The set \( L_0 \) of all 0-approximable \( \ell \) -varieties is a lattice relative to the naturally defined operations of union and intersection. Let \( V, V' \in L_0 \). Then we say that \( V \) covers \( V' \) in the lattice \( L_0 \) if \( V \supseteq V' \) and \( V \supseteq U \supseteq V \) implies \( V = U \) or \( V = U \), where \( U \in L_0 \).

In this article is proved the existence of \( 2^{\omega_0} \) 0-approximable \( \ell \) -varieties having \( 2^{\omega_0} \) distinct coverings in the lattice \( L_0 \) of 0-approximable \( \ell \) -varieties.

1. A group \( G \) equipped with a linear order \( \rho \) will be denoted by \( (G, \rho) \). As usual, \( |x| = x \land x' \), \( [x, y] = x \land y \). If \( \gamma \) is a real number, then \( [\gamma] \) is the integer part of the number \( \gamma \). Basic facts and definitions on linearly and lattice ordered groups can be found in [1-3], and on group theory in [4] and [5].

Let \( 0 \neq A_\beta \) be a subgroup of the naturally ordered additive group \( R \) of real numbers; \( \beta \) a positive real number such that \( a \in A_\beta \) implies \( \beta a, \beta^{-1} a \in A_\beta \), \( B_\beta = (\beta) \) the infinite cyclic subgroup of the multiplicative group of positive real numbers generated by the number \( \beta \).

Consider the set \( T_\beta = \{(a, a') \mid a \in B_\beta = (\beta), a \in A_\beta \} \) with the operation of multiplication \( (a, a')(a', a'') = (aa', a + a') \). We assume that \( T_\beta \cap T_\rho = (e) \) if \( \rho = \beta^p \) and \( p > 0 \) or \( p = 0 \) and \( a > 0 \) in \( A_\beta \). Then this order is linear; we denote it by \( \beta \). Note that the set of elements of the group \( T_\beta \) of the form \( (a, a') \), where \( a \in A_\beta \), is a convex invariant subgroup.

isomorphic to $A_\beta$, and $(\tau, t)(4, \omega)(\tau, t) = (4, \omega)$. We denote this subgroup by $A'_\beta$. The set of elements of $T_\beta$ of the form $(\tau, 0)$, where $\tau \in B_\beta - (\beta)$, constitutes a subgroup $B'_\beta$ isomorphic to $B_\beta$. Obviously, the group $T_\beta$ is a semidirect product of $A'_\beta$ and an infinite cyclic group $B'_\beta$ (see p. 336 in [5]).

**Lemma 1.** Let $(G, \mathcal{P})$ be a non-Abelian linearly ordered group having an archimedean invariant convex subgroup $A$ such that the quotient group $G/A$ is an infinite cyclic group. Then $(G, \mathcal{P})$ is order-isomorphic to the linearly ordered group $(T_\beta, \mathcal{P}_\beta)$ for some positive real number $\beta \neq 1$ and $0 \neq A_\beta \subset \mathbb{R}$.

The proof of Lemma 1 directly follows from the H"{o}lder theorem and the description of order-automorphisms of archimedean linearly ordered groups (see pp. 27-28 in [1]).

**Lemma 2** (p. 227 in [6]). Let $V_1$ and $V_2$ be $\ell$-varieties and $(G, \mathcal{P})$ a linearly ordered group. If $(G, \mathcal{P}) \in V_1 \cup V_2$ then $(G, \mathcal{P}) \in V_1$ or $(G, \mathcal{P}) \in V_2$.

2. Let $\gamma$ be a real number and $\gamma \geq 4$. If $\gamma = \frac{m_0}{n_0}$ is a rational number, then let $V_\gamma$ be the $\ell$-variety defined by the identities $\Sigma_\gamma$:

\begin{align*}
& a) \quad \langle [x, y]^n \rangle \cap [x, y]^n = \langle [x, y]^n \rangle, \\
& b) \quad (x \wedge y \cdot x^{-1} y) \vee e = e.
\end{align*}

If $\gamma$ is an irrational number, then $\gamma = \sup \left\{ \frac{m_i}{n_i} \right\}$, where $\frac{m_i}{n_i}$ are rational numbers such that $\frac{m_0}{n_0} < \gamma$. Let $V_\gamma$ be the $\ell$-variety defined by the system of identities $\Sigma_\gamma$:

\begin{align*}
& a) \quad \langle [x, y]^n \rangle \cap [x, y]^n = \langle [x, y]^n \rangle, \\
& b) \quad (x \wedge y \cdot x^{-1} y) \vee e = e.
\end{align*}

We denote by $V^3$ the $\ell$-variety defined by the following system of identities $\Sigma_3$:

\begin{align*}
& a) \quad \langle [x, y]^n \rangle \cap [x, y]^n = \langle [x, y]^n \rangle; \\
& b) \quad (x \wedge y \cdot x^{-1} y) \vee e = e.
\end{align*}

Let $(G, \mathcal{P})$ be a linearly ordered group in the $\ell$-variety $V^3$. Consider all possible $\omega, \sigma \in G$ such that the leap of convex subgroups $\overline{G}_\omega \supset G_\sigma$ of the linearly ordered group $(G, \mathcal{P})$ defined by the element $[\omega, \sigma] = e$ is invariant relative to conjugation by the element $[\omega, \sigma]$. Then in the subgroup $H_{\omega, \sigma} = g \{ ([\omega, \sigma] \cap G_\sigma) \}$, linearly ordered relative to the induced group $\mathcal{P}_{\omega, \sigma}$, the subgroup $G_\sigma$ is convex and invariant.

Consider the quotient group $\overline{H}_{\omega, \sigma} = H_{\omega, \sigma} / G_\sigma$, naturally linearly ordered by the linear order $\overline{\mathcal{P}}_{\omega, \sigma}$. Since $(G, \mathcal{P}) \in V^3$, $\overline{H}_{\omega, \sigma}$ is non-Abelian. The subgroup $\overline{A} = \overline{G}_\sigma / G_\sigma$ is an archimedean invariant convex subgroup in $\overline{H}_{\omega, \sigma}$, and the quotient group $\overline{H}_{\omega, \sigma} / \overline{A}$ is an infinite cyclic group. Therefore, by Lemma 1, $(\overline{H}_{\omega, \sigma}, \overline{\mathcal{P}}_{\omega, \sigma})$ is order-isomorphic to the linearly ordered group $(T_{\beta_\omega, \sigma}, \mathcal{P}_{\beta_\omega, \sigma})$ for some positive real number $\beta(\omega, \sigma) \neq 1$ and $0 \neq A_{\beta_\omega, \sigma} \subset \mathbb{R}$.