COROLLARY 2 [7]. \( \text{Th}(L_2) \neq \text{Th}(L_\rho) \) for \( \rho \in \{d, \rho, \emptyset\} \).

LITERATURE CITED


COVERINGS IN THE LATTICE OF \( \emptyset \)-VARIETIES

N. Ya. Medvedev

An \( \emptyset \)-variety \( V \) in which the identity \((x \vee y) \wedge x \wedge y \vee e = e\) is valid is said to be a 0-approximable \( \emptyset \)-variety. The set \( L_0 \) of all 0-approximable \( \emptyset \)-varieties is a lattice relative to the naturally defined operations of union and intersection. Let \( \overline{V} \subseteq V \subseteq L_0 \). Then we say that \( \overline{V} \) covers \( V \) in the lattice \( L_0 \) if \( \overline{V} \supseteq V \) and \( \overline{V} \supseteq U \supseteq V \) implies \( \overline{V} = U \) or \( V = U \), where \( U \in L_0 \).

In this article is proved the existence of \( 2^{\aleph_0} \) 0-approximable \( \emptyset \)-varieties having \( 2^{\aleph_0} \) distinct coverings in the lattice \( L_0 \) of 0-approximable \( \emptyset \)-varieties.

1. A group \( G \) equipped with a linear order \( \rho \) will be denoted by \( (G, \rho) \). As usual, \( |x| = x^+x^- \), \( [x, y] = x^+y^- \). If \( \gamma \) is a real number, then \( [\gamma] \) is the integer part of the number \( \gamma \). Basic facts and definitions on linearly and lattice ordered groups can be found in [1-3], and on group theory in [4] and [5].

Let \( 0 \neq A_\beta \) be a subgroup of the naturally ordered additive group \( R \) of real numbers; \( \rho \neq \beta \) a positive real number such that \( \alpha \in A_\beta \) implies \( \beta\alpha, \beta^{-1}\alpha \in A_\beta \), \( B_\beta = (\beta) \) the infinite cyclic subgroup of the multiplicative group of positive real numbers generated by the number \( \beta \).

Consider the set \( T_\beta = \{(\alpha, \alpha) \mid \alpha \in B_\beta = (\beta), \alpha \in A_\beta \} \) with the operation of multiplication \( (\tau \alpha, \alpha') = (\tau \alpha, \alpha + \alpha') \). We assume that \( T_\beta \theta (\alpha, \alpha) \gg e \) if \( \tau = \beta^p \) and \( \rho > 0 \) or \( \rho = 0 \) and \( \alpha > 0 \) in \( A_\beta \). Then this order is linear; we denote it by \( \theta_\beta \). Note that the set of elements of the group \( T_\beta \) of the form \( (\alpha, \alpha) \), where \( \alpha \in A_\beta \), is a convex invariant subgroup.
isomorphic to $A_{\mathfrak{A}}$, and $(\tau, t) \cdot (t, \alpha) \cdot (\tau, t)^{-1} = (t, \gamma \alpha)$. We denote this subgroup by $A'_{\mathfrak{A}}$. The set of elements of $T_{\mathfrak{A}}$ of the form $(\tau, 0)$, where $\tau \in B_{\mathfrak{A}} = (\beta)$, constitutes a subgroup $B'_{\mathfrak{A}}$ isomorphic to $B_{\mathfrak{A}}$. Obviously, the group $T_{\mathfrak{A}}$ is a semidirect product of $A'_{\mathfrak{A}}$ and an infinite cyclic group $B'_{\mathfrak{A}}$ (see p. 336 in [5]).

**LEMMA 1.** Let $(G, \mathcal{P})$ be a non-Abelian linearly ordered group having an archimedian invariant convex subgroup $A$ such that the quotient group $G/A$ is an infinite cyclic group. Then $(G, \mathcal{P})$ is order-isomorphic to the linearly ordered group $(T_{\mathfrak{A}}, \mathcal{P})$ for some positive real number $\beta \neq 1$ and $0 \neq A_{\mathfrak{A}} \subseteq R$.

The proof of Lemma 1 directly follows from the Hölder theorem and the description of order-automorphisms of archimedian linearly ordered groups (see pp. 27-28 in [1]).

**LEMMA 2** (p. 227 in [6]). Let $V_1$ and $V_2$ be $l$-varieties and $(G, \mathcal{P})$ a linearly ordered group. If $(G, \mathcal{P}) \in V_1 \cup V_2$ then $(G, \mathcal{P}) \not\in V_1$ or $(G, \mathcal{P}) \not\in V_2$.

2. Let $\gamma$ be a real number and $\gamma \geq 4$. If $\gamma = \frac{m}{n}$ is a rational number, then let $V^{\gamma}$ be the $l$-variety defined by the identities $\Sigma_{\gamma}$:

   a) $\langle (x, y) \rangle^{\gamma} \cdot (x, y)^{-1} \langle x, y \rangle^{\gamma} = (m, n > 0)$;

   b) $(x, y, x^{-1} y) \vee e = e$.

If $\gamma$ is an irrational number, then $\gamma = \sup \left\{ \frac{m_i}{n_i} \mid i \in I \right\}$, where $\frac{m_i}{n_i}$ are rational numbers such that $q < \frac{m_i}{n_i} < \gamma$. Let $V^\gamma$ be the $l$-variety defined by the system of identities $\Sigma_{\gamma}$:

   a) $\langle (x, y) \rangle^{\gamma} \cdot (x, y)^{-1} \langle x, y \rangle^{\gamma} = (m, n > 0)$;

   b) $(x, y, x^{-1} y) \vee e = e$.

We denote by $V^3$ the $l$-variety defined by the following system of identities $\Sigma_3$:

   a) $\langle (x, y) \rangle^{3} \cdot (x, y)^{-1} \langle x, y \rangle^{3} = (m, n > 0)$;

   b) $(x, y, x^{-1} y) \vee e = e$.

Let $(G, \mathcal{P})$ be a linearly ordered group in the $l$-variety $V^3$. Consider all possible $u, v \in G$ such that the leap of convex subgroups $G_{u,v} \rightarrow G_{u}$ of the linearly ordered group $(G, \mathcal{P})$ defined by the element $[u, v] \neq e$ is invariant relative to conjugation by the element $[u, v, u, v]$. Then in the subgroup $H_{u, v} = \langle ([u, v]^{1}, G_{u}) \rangle$, linearly ordered relative to the induced group $P_{u, v}$, the subgroup $G_{u}$ is convex and invariant.

Consider the quotient group $\overline{H}_{u, v} = H_{u, v} / G_{u}$, naturally linearly ordered by the linear order $P_{u, v}$. Since $(G, \mathcal{P}) \in V^3$, $\overline{H}_{u, v}$ is non-Abelian. The subgroup $\overline{A} = \overline{G}_{u} / G_{u}$ is an archimedian invariant convex subgroup in $\overline{H}_{u, v}$, and the quotient group $\overline{H}_{u, v} / \overline{A}$ is an infinite cyclic group. Therefore, by Lemma 1, $(\overline{H}_{u, v}, \overline{P}_{u, v})$ is order-isomorphic to the linearly ordered group $(T_{\mathfrak{A} u, v}, \mathcal{P}_{\mathfrak{A} u, v})$ for some positive real number $\beta (u, v) \neq 1$ and $0 \neq A_{\mathfrak{A} u, v} \subseteq R$. 

40