ON THE STRESSES
PRODUCED BY IMPULSIVE DISPLACEMENTS APPLIED
TO THE INNER SURFACE OF A SPHERICAL CAVITY

by Ghasi Ram Verma (*)

Summary — In this note displacements and stresses produced in an infinite elastic solid by impulsive radial and twisting displacements applied to the inner surface of a spherical cavity have been determined.

Introduction — Problems of radial and twisting impulsive forces acting on the inner surface of a spherical cavity in an isotropic infinite medium have been recently solved in an elegant manner by Das Gupta (1954). The object of this note is to solve the corresponding problems when impulsive displacements instead of impulsive forces are prescribed.

1. Impulsive Radial Displacement — The origin of coordinates is taken at the centre of the cavity whose surface is given by the equation \( r = c \) (constant) where \( r^2 = x^2 + y^2 + z^2 \). Assuming that there is radial displacement \( U(r,t) \), we have the displacement components in Cartesian coordinates as

\[
\begin{align*}
(u, v, w) &= \left( \frac{x}{r}, \frac{y}{r}, \frac{z}{r} \right) U(r, t) \\
&= \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \Phi(r, t),
\end{align*}
\]

if

\[
\frac{\partial \Phi(r, t)}{\partial r} = U(r, t).
\]

From the equations (1.1) we obtain

\[
\Delta = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \nabla^2 \Phi.
\]

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The equations of motion

\[ (\lambda + \mu) \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \Delta + \mu \nabla^2 (u, v, w) = \rho \frac{\partial^2}{\partial t^2} (u, v, w) \]

give

\[ (\lambda + 2\mu) \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \nabla^2 \Phi = \rho \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \frac{\partial^2 \Phi}{\partial t^2} \]

These equations will be satisfied if \( \Phi \) satisfies the equation

\[ \nabla^2 \Phi = \frac{1}{a^2} \frac{\partial^2 \Phi}{\partial t^2} , \]

where

\[ a^2 = \frac{\lambda + 2\mu}{\rho} . \]

As \( \Phi \) is a function of \( r \) and \( t \) only, the equation (1.3) reduces to

\[ \frac{\partial^2}{\partial r^2} (r \Phi) = \frac{1}{a^2} \frac{\partial^2}{\partial t^2} (r \Phi) \]

of which the solution can be put in the form

\[ \Phi = \frac{f \left( t - \frac{r-c}{a} \right)}{r} , \]

the reflected wave being neglected since the surrounding medium is supposed to extend to infinity.

The above value of \( \Phi \) gives

\[ U = -\frac{1}{ar} f' \left( t - \frac{r-c}{a} \right) - \frac{1}{r^2} f \left( t - \frac{r-c}{a} \right) , \]

where dashes denote differentiation with respect to \([t - (r-c)/a]\). For impulsive radial displacement on the surface of the cavity we assume

\[ U = u_0 \delta(t) , \]

where \( u_0 \) is constant and \( \delta(t) \) is Dirac's delta function (cf. Carslaw & Jaeger, art. 106).

Thus we obtain from (1.5)

\[ u_0 \delta(t) = -\frac{1}{ac} f'(t) - \frac{1}{c^2} f(t) . \]