EFFICIENTLY UPDATING CONSTRAINED DELAUNAY TRIANGULATIONS

CAO AN WANG

Department of Computer Science, Memorial University of Newfoundland, St. John's, Newfoundland, Canada A1C 5S7

Abstract.

The Constrained Delaunay Triangulation of a set of obstacle line segments in the plane is the Delaunay triangulation of the endpoint set of these obstacles with the restriction that the edge set of the triangulation contains all these obstacles. In this paper we present an optimal \( \Theta(n + k) \) algorithm for inserting an obstacle line segment or deleting an obstacle edge in the constrained Delaunay triangulation of a set of \( n \) obstacle line segments in the plane. Here \( k \) is the number of Delaunay edges deleted and added in the triangulation during the updates.

CR Categories: F 2.1.

Key Words: Constrained Delaunay Triangulation, Delaunay-Ordered Polygon, Voronoi Diagram.

1. Introduction.

Delaunay triangulation and its dual, Voronoi diagram, are two important data structures in computational geometry. The two structures for a set of points (called sites) have been extensively studied [3, pp. 198–218, 12]. Motivated by geographical interpolation problems Lee and Lin [7] first investigated Delaunay triangulation in the presence of obstacles. They proposed an \( O(n^2) \) algorithm for finding the constrained (which they called generalized) Delaunay triangulation of a set of \( n \) line segments as well as an \( O(n \log n) \) algorithm for the triangulation of a simple polygon, where the endpoints of the line segments and the vertices of the polygon are regarded as sites and the line segments and the edges are regarded as obstacles. Later, the time bound for solving the problem in the first case was reduced to \( \Theta(n \log n) \) [4, 13, 14]. However, whether or not the problem in the second case can be solved in \( O(n \log n) \) time is not known [1]. This outstanding open problem for the case of a convex polygon has been solved by Aggarwal et al. [2], but the problem for the case of a general simple polygon still remains open. Recently, a linear-time algorithm for

---

\(^{1}\) This work is supported by NSERC grant OPG0041629.

the problem of a monotone histogram has been presented by Djidjev and Lingas [5], and thus the problem of a general simple polygon can be solved in $O(n \log r)$ time, where $r$ is the number of reflex angles of the polygon. In this paper, we show a linear-time algorithm for the problem in the case of a special simple polygon, called the Delaunay-Ordered polygon. Moreover, the duality of constrained Delaunay triangulation and constrained Voronoi diagram of a set of line segments is studied in [6].

For updating, that is inserting or deleting a site in the Voronoi diagram of a set of $n$ sites, Aggarwal et al. [2, p. 601] proposed an optimal $\Theta(\log n + k)$ method for deleting a site in the diagram, where $\log n$ is the time for point location and $k$ is the number of Voronoi edges deleted and added during the update process. Since inserting a site in the diagram takes $\Theta(\log n + k)$ time, the diagram can be updated in $\Theta(\log n + k)$ time. By the duality, the corresponding Delaunay triangulation can be updated in the same time bound. However, whether or not constrained Delaunay triangulations and constrained, or bounded, Voronoi diagrams can be updated efficiently is not known. Directly using existing algorithms [4, 13, 14] for updates is inefficient because these algorithms are required to rebuild the entire triangulation. In this paper, we present an optimal $\Theta(\log n + k)$ time update algorithm.

For simplicity, we use $CDT$ and $C\text{Vor}$ to denote the terms: constrained Delaunay triangulation and constrained Voronoi diagram respectively.

2. Finding the $CDT$ of a special polygon.

In Definitions 1 to 4, we shall define constrained Delaunay triangulation and its dual, extended constrained Voronoi diagram. Let $L$ denote a set of non-intersecting line segments (except possibly at their endpoints) representing obstacles. Let $S$ denote the endpoint set of $L$.

**Definition 1:** The distance of $s \in S$ and $x \in \mathbb{R}^2$ in the presence of obstacles is determined by

$$d_L(x, s) = \begin{cases} d(x, s) & \text{if } x \text{ and } s \text{ are visible from each other,} \\ \infty & \text{otherwise.} \end{cases}$$

**Definition 2:** The $C\text{Vor}$ of $L$, denoted by $C\text{Vor}(L)$, is a set of Voronoi cells $\{V(s) \mid s \in S\}$, where $V(s) = \{x \in \mathbb{R}^2 \mid d_L(x, s) \leq d_L(x, s') \text{ and } d_L(x, s) \neq \infty, \text{ for all } s' \in S, s \neq s'\}$.

The boundary of a Voronoi cell $V(s)$ is the closure of $V(s)$. A Voronoi edge is a maximal straight line segment on the boundary of a Voronoi cell. A Voronoi vertex is an endpoint of a Voronoi edge. The $C\text{Vor}$ of a simple polygon is a special case of $C\text{Vor}(L)$ that $L$ is a simple polygon.