THE DETERMINATION OF THE REGIONAL PART OF THE VERTICAL GRADIENT ANOMALY BY A GEODETIC METHOD

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Summary — The significance of the vertical gradients of gravity is great in geophysics and also in geodesy. In geophysics the observed vertical gradients can give valuable information about mass distributions close to the surface of the earth and in geodesy they may be used in determining the shape of the equipotential surfaces. The observed vertical gradients are very sensitive to masses close to the surface of the earth and they change very rapidly. Therefore, they should not be used for purposes such as the reduction of the observed gravity to the sea level. The normal vertical gradients are not the best either for this purpose because they are much too uniform on the surface of the earth. The best values for practical purposes are probably the regional vertical gradients.

This paper presents a method to determine the regional vertical gradient anomalies in large areas from geodetic observations which, added to the normal part, will give the regional vertical gradients.

In geodetic and gravimetric computations the coordinate system generally used is a local coordinate system. The x axis at such a system is oriented towards the North, i.e., it is tangent to the local meridian; the z axis coincides with the direction of the local gravity; and the y axis is perpendicular to both x and z and it is positive towards East. Therefore, the relative orientation of the coordinate system is different at each observation station. For the sake of simplicity let us use only one coordinate system for a small area.

We choose an arbitrary station, O, with astro-coordinates, $\phi$ and $\lambda$, as our reference station and we use the coordinate system based on this point over the whole area. Hence at any point the $z$ axis is parallel to the vertical at point $O$ and the $x$ axis is parallel to the tangent to the meridian at point $O$.

Figure 1 shows the $xz$ plane of a coordinate system as defined above and through an arbitrary point, $A_1$. The origin of the system is at $A_1$. The $z$ axis is parallel to the vertical at the reference station $O$; therefore, it does not coincide with the direction of the gravity vector $g$ at point $A_1$. The vector $A_1G$ is the projection of the gravity vector $g$ on to the $xz$ plane. The difference between the vector $A_1G$ and the gravity vector is negligible. In this case the $A_1H$ vector is the $g_x$ compo-

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ponent of \( g \). The line \( E - E \) is the intersection of the equatorial plane and the \( xz \) plane. If the astro-latitude of the reference station is \( \Phi \) and the angle between the vertical at \( O \) and at \( A_1 \) is \( \Delta \Phi_1 \), then the latitude at \( A_1 \) is \( \Phi_1 = \Phi + \Delta \Phi_1 \). From Figure 1 it is obvious that

\[
g_x = g \sin \Delta \Phi_1
\]
or since \( \Delta \Phi_1 \) is a small angle

\[
\Delta \Phi_1 = -\frac{g_x}{g}.
\]

After a similar approach the angle difference in the \( yz \) plane, i.e., in the prime-vertical plane is

\[
\Delta L_1 \cos \Phi_1 = -\frac{g_y}{g}
\]
where \( \Delta L_1 \) is the astro-longitude difference between the reference station \( O \) and \( A_1 \).

Expressions (1) and (2) give the North and the East components of the angle between the true verticals at \( O \) and at \( A_1 \) respectively. We can similarly determine the North and East components of the angle between the verticals at \( O \) and another arbitrary point, \( A_2, \Delta \Phi_2 \) and \( \Delta L_2 \). The quantities \( (\Delta \Phi_2 - \Delta \Phi_1) \) and \( (\Delta L_2 - \Delta L_1) \) are the North and the East components of the angle between the true normals at \( A_2 \) and at \( A_1 \).

\[
\begin{align*}
\Delta \Phi_2 - \Delta \Phi_1 &= -\frac{1}{g_m} [(g_x)_2 - (g_x)_1] = -\frac{1}{g_m} \left[ \left( \frac{\partial W}{\partial x} \right)_2 - \left( \frac{\partial W}{\partial x} \right)_1 \right] \\
(\Delta L_2 - \Delta L_1) \cos \Phi_m &= -\frac{1}{g_m} [(g_y)_2 - (g_y)_1] = -\frac{1}{g_m} \left[ \left( \frac{\partial W}{\partial y} \right)_2 - \left( \frac{\partial W}{\partial y} \right)_1 \right]
\end{align*}
\]
where \( g_m \) and \( \Phi_m \) are the mean gravity and the mean astro-latitude between \( A_1 \) and \( A_2 \). The quantities \( (\Delta \Phi_2 - \Delta \Phi_1) \) and \( (\Delta L_2 - \Delta L_1) \) are the astro-latitude and longitude differences between points \( A_2 \) and \( A_1 \). The \( (g_x)_2, (g_x)_1, \) etc. are the gravity components in the \( x \) or \( y \) directions at \( A_1 \) and \( A_2 \). \( W \) is the potential function of the gravity (geopotential function).