Summary — Equations for the motions in a galaxy which is controlled by gravitational forces and inertial effects are formulated. It is found that for the motions relative to the rotating system formulae analogous, and of similar form, to the geostrophic wind equations may be written. From these formulae and from the distribution of the relative gravitational potential in the disc of the galaxy, it is found that a spiral tendency in the mass distribution carries the implication of an inward flux of angular momentum by advective processes. This is to be compared with an outward flux through gravitational torques obtained in previous studies in which the writer participated.

1. Introduction — A large tract of Newtonian theory can be more compactly and conveniently discussed in terms of the gravitational potential \( \varphi \), a concept introduced subsequent to the time of Newton. The fundamental equation relating \( \varphi \) to the spacial distribution of mass is the familiar equation of Poisson, namely,

\[
\nabla^2 \varphi = 4\pi G \rho
\]

where \( G \) is the universal constant of gravitation and \( \rho \) is the density of mass in space.

It is well known that certain philosophical difficulties arise in an attempt to apply (1) to the entire universe as was pointed out notably by Einstein (1917), and modern cosmological theory may be said to have had its beginning in this source. If the mass density \( \rho \) is taken over some finite region of space, one property of equation (1) is that it alone does not possess the competence to specify the motion of a given interior mass point. The solution of (1), speaking mathematically, consists of two parts, namely, a complimentary function which is not uniquely fixed by the interior mass, and a particular integral which is so determined. For the specification of the complimentary function one must apply as a boundary condition added information, let us say, concerning the value of the potential at an infinite distance from the given mass distribution if it is one that is isolated in space, as we shall for our present purpose now assume. Under these conditions the subject takes on a simple form owing to the fact that the complimentary function, since it is a solution of (1) with the right hand member zero (i.e., of Laplace's

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equation), must be an harmonic function. Its vanishing over a sphere of infinite radius necessitates its identical vanishing at all interior points, so that only the particular integral remains to be considered. This integral is

$$\varphi(x, y, z) = -G \int \int \int \frac{\varphi' dx'dy'dz'}{\sqrt{(x-x')^2 + (y-y')^2 + (z-z')^2}}$$

when cartesian coordinates \((x, y, z)\) are used. The force \(\vec{F}\) on a mass \(\varphi\) per unit volume is now given by

$$\vec{F} = -\varphi \nabla \varphi$$

and, in the absence of other forces, the equation of motion is simply,

$$\frac{d\vec{C}_0}{dt} = -\nabla \varphi$$

where \(\vec{C}_0\) is the velocity relative to a so-called inertial coordinate system whose use is here for the present presupposed and \(t\) is time. In equations (1), (2) the constant factor \(G\) has been retained explicitly, instead of being absorbed in \(\varphi\), in order not to lose track of its role in the problem.

Although we shall not make it our purpose now to consider the points at which the theory outlined is insufficient when applied to the universe as a whole, it seems quite plausible to apply it to a single galaxy, as an approximation in order to study its internal motions in a gross fashion. For this purpose the dismissal of other galaxies is but an artifice to secure a first result, subject to possible later amendment. In these further considerations the galaxy will be assumed to have its disc structure in the \(x, y\)-plane and its principal axis passing through the origin along the \(z\) coordinate. The halo of Population II stars and globular clusters will be neglected.

Due to the remarkable thinness of the disc (see, e.g., OORT, KER & WESTERTHOUT, 1958), its mass distribution and motion may be approximated two-dimensionally. Thus we may define

$$\varphi_1 (x, y) = \int_{-\infty}^{+\infty} \varphi (x, y, z) \, dz.$$ 

Since the motions are assumed to take place essentially in the \(x, y\)-plane, it is seen that, although \(\varphi = \varphi (x, y, z)\) for the potential of the disc, interest focuses on the distribution of \(\varphi\) along the \(x, y\)-plane. Therefore, we may rewrite (2) with the aid of (5) in the approximate form

$$\varphi_0 \approx -G \int \int \frac{\varphi_1' dx'dy'}{\sqrt{(x-x')^2 + (y-y')^2}}$$

where \(\varphi_0\) is the value of \(\varphi\) at \(z = 0\).

Within the same general standard of approximation one may rewrite (3) and (4) as follows

$$\vec{F}_1 \approx \varphi_1 \nabla \varphi_0$$