WAVE PROPAGATION IN MARINE SEDIMENTS AND WATER SATURATED SOILS

by S. K. Bose (*)

Summary — A unification of the theories of Biot and Weiskopf has been made to form the suitable equations of motion for porous water saturated soils and marine sediments. It has been shown that the velocities of the body waves depend on the direction of propagation. In the vertical direction there are three, one distortional and two dilatational waves. In the horizontal direction there are two dilatational and two distortional waves. Finally, propagation of Love waves and Rayleigh waves have been discussed. Suitable potential functions have been derived to find the frequency equation for Rayleigh waves.

In recent years, considerable experimental work has been done to study the physical, chemical and seismic properties of marine sediments [cf. HAMILTON & al. (1956), LOUGHTON (1954), (1957), SUTTON & al. (1957), NAPE & DRAKE (1957)], as also for soils [cf. TERZAGHI (1943), TERZAGHI & PECK (1948), TAYLOR (1948), TSCHEBOTARIOFF (1951), HEILAND (1946), KAILASAM (1953)]. Theoretical models have also been proposed to fit and predict observational data. For the Sediments the semi-empirical formulae of WOOD (1941) and NAPE & DRAKE (1957) are in much use. The theories of URICK (1947, 1948), URICK & AMENT (1949), AMENT (1953), which assume the absence of shear strength of the medium, have also been used. The more complete theory of BIOT (1956) for wave propagation in a statistically isotropic fluid saturated porous solid, which includes the shear strength of the medium has been used little due to the difficulty of evaluating the material constants involved in the equations.

As for the soils the most satisfactory theory is that given by WEISKOFF (1945), which considers the medium to have a particular kind of transverse-isotropy. The only defect of this theory is that it considers the solid medium to be continuous and not porous.

That the marine sediments have rigidity and anisotropy have been shown by LOUGHTON (1957) and SUTTON & al. (1957). The effect of rigidity on seismic waves have been discussed by OLIVER & DORMAN (1961). Therefore for a better theoretical model both these aspects should be taken into account. As for the nature of anisotropy, we notice that the sediments are under a considerably large hydrostatic pressure viz. 400 kg/cm² and deposit in horizontal layers at the rate of approximately

(*) SUJIT KUMAR BOSE, Presidency College, Calcutta-12, India.
1 cm in a 1000 years or less (LOUGHTON 1957). Thus, we may as a first approximation consider the medium to have symmetry about vertical lines, although due to slow rate of compaction, secondary effects may set in. As for the soils, the difficulty can be removed if the more recent theory of Oshima (1954, 1955) for wave propagation in granular medium is modified to include anisotropy. This will not be done here. On the other hand we shall consider water saturated soils as is the case with Gangatic alluvium. In this way we will have a unified theory for both the media.

Here a unification of the theories of Biot (1956) and Weiskopf (1945) will be done in order to have a closer approximation to the physical state of the medium without introducing too many complications. Two assumptions have been made. Firstly, the medium is supposed to have kinetic isotropy, but elastic anisotropy of the Weiskopf-type. Secondly, the viscosity of water is omitted due to the fact that very little dispersion of the waves is observed. It has been found that the velocity of body waves depends on the direction of propagation. The velocities of the fast and slow P-waves in the x, y, and z directions are the same.

There are two S-waves propagating in the x or y directions and only one in the z-direction, as in the case of ordinary solids. The Love-waves are affected to the extent of density only. The Rayleigh waves are affected to a great extent due to the presence of the water and its velocities are given by a thirty degree equation. Jones (1961) has demonstrated the existence of two Rayleigh waves in an isotropic fluid saturated solid when there is no kinetic coupling between the fluid and the solid. Here the number of Rayleigh waves could not be ascertained theoretically. For this the help of numerical methods might have been taken, but for the lack of pertinent data.

1. Formulation of the basic equations.

Consider a water saturated porous medium with kinetic isotropy and anisotropy of the Weiskopf-type. If the viscosity of the water is neglected, the equations of motion in the absence of body forces are of the types [Biot (1956)]

\[
\begin{align*}
\frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{xy}}{\partial y} + \frac{\partial T_{xz}}{\partial z} &= \frac{\partial^2}{\partial t^2} (\rho_{11} u + \rho_{12} U), \\
\frac{\partial T}{\partial x} &= \frac{\partial^2}{\partial t^2} (\rho_{12} u + \rho_{22} U)
\end{align*}
\]

where \(T_{xx}, T_{xy}, \ldots\) are the components of the stress tensor on the solid part of the medium and the scalar \(T\) is related to the fluid pressure \(p\) according to the relation

\[-T = fp\]

where \(f\) is the porosity of the medium. \((u, v, w)\) and \((U, V, W)\) are the components of the displacement vector of the solid and fluid part respectively.

The kinetic coefficients \(\rho_{11}, \rho_{22}, \rho_{12}\) are connected with the densities \(\rho, \rho_s, \rho_w\) of the medium, solid and water respectively according to the relations

\[
\begin{align*}
\rho_{11} + \rho_{12} &= (1 - f) \rho_s \\
\rho_{12} + \rho_{22} &= f \rho_w
\end{align*}
\]