Heat transfer in an MHD channel flow with boundary conditions of the third kind

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Abstract. The heat transfer equation for a two-dimensional magnetohydrodynamic channel flow has been solved using boundary conditions of the third kind considering a discontinuity in the “ambient” temperature. The boundary conditions of the third kind indicate that the normal temperature gradient at a particular point in the boundary is assumed to be proportional to the difference between the fluid temperature and the externally prescribed ambient temperature. The presence of an external circuit is also considered to permit the flow of an electric current in the direction perpendicular to the plane of analysis. The resistance of the external circuit is varied from zero (closed circuit) to infinity (open circuit). Temperature fields far away from and near to the discontinuity are found separately and then added in order to obtain the temperature in the whole flow region. The solutions in the limits where the boundary conditions become first (Dirichlet) or second (Neumann) kind are discussed and the influence of the external resistance and the Hartmann, Péclet and Biot numbers on the temperature distribution is investigated.

1. Introduction

Magnetohydrodynamic (MHD) flows were studied intensively in the sixties when MHD generators were built as promising alternative energy converters. Interest declined in most parts of the world before a definite assessment on the performance and possibilities of such devices could be established. However, interest has been renewed in recent years mostly due to several technological applications. One of them is an electric generator designed with liquid metal as working fluid and using solar energy as the primary power source. A few prototypes demonstrating the feasibility have been built [8]. A second important application of these flows concerns the cooling systems of fusion reactors where liquid lithium moves in presence of strong magnetic fields, generating MHD flows that are hitherto not satisfactorily described [9].

In the present paper we address the problem of heat transfer inside a duct where a conducting fluid flows in the presence of a transverse magnetic field. Due to Joule effect and viscous dissipation, the system presents internal heat sources; this heat must be removed through the walls in order to prevent the fluid temperature from rising to unacceptable values where phase change with the corresponding pressure rise, may occur. The amount of heat entering or leaving the system depends on the external or ambient temperature as well as on the wall heat transfer coefficient. This condition is known as boundary condition of the third kind and is regarded as more realistic than
considering the wall temperature of the duct as constant (Dirichlet or first kind conditions) insofar the heat transfer from the duct to the surroundings is taken into account via a “Newton’s cooling law”. Strictly speaking, the heat transfer to the surroundings should be calculated by solving the non-isothermal flow outside the duct. Such situations known as conjugate heat transfer problems are vastly more complicated and for the present we assume that a model using a Newton’s cooling law approach is satisfactory.

The present work is based on the method proposed by Singh [10] and Nigam and Singh [6] to solve a class of equations that include the heat conduction equation for an MHD flow in a duct with prescribed wall temperatures (boundary conditions of the first kind). This method has been used by several authors to tackle related problems. In [6] the Joule-heating term was wrongly considered; this point was corrected in [7] where the case with constant wall heat flux was analyzed. Jain [3] solved the same problem studied in [6] but considering electrically-conducting walls. A similar problem in a duct formed by coaxial cylinders was analyzed by Singh and Lal [11]. The same authors worked on a 3D MHD flow in a rectangular duct using a numerical method [12]. The studies by Javeri [4] and [5] deserve a special comment since the MHD heat transfer equation is solved using boundary conditions of the third kind. In [4] Javeri considered the heat transfer in the thermal entrance region of a flat MHD channel but all heat dissipation effects and axial heat conduction are neglected. The one modification of the model studied in [5] is the incorporation of the Hall effect and ion slip. In the present study, the heat dissipation due to Joule and viscous effects are taken into account. Likewise, axial heat conduction and the possibility of a net electrical current flowing through an external resistance are considered. The external ambient temperature is held constant up to a certain location where it changes to another constant value; this discontinuity in the ambient temperature introduces a perturbation in the temperature field of the flow region which has little influence at large distances from this location. The influence of the different physical parameters on the temperature distribution is studied by varying the external resistance and the Hartmann, Péclet and Biot numbers.

2. Analysis

2.1. Problem formulation

Consider the steady-state laminar flow of a viscous incompressible fluid with non-zero electrical conductivity inside a duct and in presence of a uniform magnetic field $B_0$ oriented in the vertical direction. The distance between the side walls of the channel is much larger than that separating the top and bottom walls ($a$) and therefore, the flow is considered two-dimensional. The situation considered is shown schematically in Fig. 1.

The channel is considered infinite in the $x'$ direction and the variables in the