A DISCRETE MODEL WITH INTERACTION BETWEEN THE BUDWORM AND ITS PREDATOR IN A CIRCULAR REGION

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Abstract

We consider a discrete model with interaction between the budworm and its predator in a circular region. The number and properties of steady solutions, and the asymptotic behavior of unsteady solutions are discussed.

I. Introduction

Ludwing, Jones, Holling studied spatially uniform budworm density in the forest. Ludwig, Aronson, Weinberger and Guo Ben-yu, Sleeman, Mitchell considered the spatial effect. On the other hand, Weinberger, Guo Ben-yu, Mitchell and Chen Sui-yang, Guo Ben-yu considered the discrete problems. This paper is devoted to the budworm population with the interaction between the budworm and its predator in a circular region.

Let $\Omega$ be a bounded open domain in $\mathbb{R}^2$ with boundary $\partial \Omega = \partial \Omega \cup \partial \Omega$ and $U(x, t)$ the sealed density of the budworm. If the exterior is lethal, then $U(x, t)$ satisfies

\[
\frac{\partial U}{\partial t} - \Delta U = f(U), \quad x \in \Omega, \ t > 0,
\]

\[
U(x, t) = 0, \quad x \in \partial \Omega, \ t > 0,
\]

\[
U(x, 0) = U_0(x), \quad x \in \Omega,
\]

where

\[
f(U) = U - \frac{U^2}{Q} - \frac{U^2}{R(1+U^2)}, \quad R > 0, \ Q > 0.
\]

The parameter $R$ is the foliage density, whereas the parameter $Q$ depends on the interaction between the budworm and its predator.

Proposition 1.1. If $Q > 3\sqrt{3}$, then there are two positive numbers $R_1(Q) < R_2(Q) < \frac{3\sqrt{3}}{8}$. If $0 < R < R_1(Q)$, then the equation $f(u) = 0$ has one positive root $u_2(Q)$. However, if $R_1(Q) < R < R_2(Q)$ then there are three positive roots $u_1(Q) < u_2(Q) < u_3(Q)$. If $R > R_2(Q)$, then there is a unique positive root $u_4(Q)$. For simplicity $u_4(Q)$ is denoted by $u_4$.

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Proposition 1.2. Let \( u_2(Q, R) \) be the smallest positive root of \( f(u) = 0 \) and \( \alpha(Q) = \max_{0 < R < R(Q)} u_2(Q, R) \). Then \( \alpha(Q) \) decreases from \( \sqrt{3} \) to 1, as \( Q \) increases from \( 3\sqrt{3} \) to \( +\infty \).

In this paper we always assume \( Q > 3\sqrt{3} \). If \( \Omega \) is a circular domain surrounding the origin with radius \( l \), \( \rho = |x| \) and \( U_0(x) = U_0(\rho) \), then

\[
\begin{align*}
\partial_U + sP_\rho = f(U), & \quad 0 < \rho < 1, \quad t > 0, \\
\partial_U \partial_\rho (0, t) = 0, & \quad U(1, t) = 0, \quad t > 0, \\
U(\rho, 0) = U_0(\rho), & \quad 0 < \rho < 1,
\end{align*}
\]

where \( s = \frac{1}{\sqrt{v}}, \quad P = -\frac{\partial^2}{\partial \rho^2} - \frac{1}{\rho} \frac{\partial}{\partial \rho} \). The corresponding steady problem is

\[
\begin{align*}
sPU - f(U) = 0, & \quad 0 < \rho < 1, \\
\partial_U \partial_\rho (0) = 0, & \quad U(1) = 0.
\end{align*}
\]

We study the fully discrete problems related to (1.2) and (1.3). We consider the existence of positive solutions of the steady problem and the asymptotic behaviour of the unsteady solution. Finally we prove the convergence of the approximate solution.

II. Difference Schémé

Let \( h = \frac{1}{N} \) and \( \tau \) be the mesh sizes of the space and time respectively. Define

\[
\begin{align*}
\Omega_h &= \{\rho = jh, \quad 1 \leq j \leq N - 1\}, \\
\Omega_h &= \{0, 1\}, \\
\mathcal{D}_h &= \Omega_h \cup \partial \Omega_h.
\end{align*}
\]

Let \( \eta^k(\rho) \) be the value of function \( \eta \) at the point \( \rho = jh \) and \( t = k\tau \). Let \( \eta^k_+(\rho) \), \( \eta^k_-(\rho) \) and \( \eta^k_0(\rho) \) denote respectively the forward, backward and central difference quotient of \( \eta^k(\rho) \) with respect to \( \rho \). \( \eta^k_+(\rho) \) denotes the forward difference quotient of \( \eta^k(\rho) \) with respect to \( t \). Let \( 0 < a < 1 \) and \( D_h(s, a) \) be a difference operator defined as

\[
D_h(s, a) u^k(\rho) = u^k(\rho) + \frac{s}{2} P_h[u^k(\rho) + u^{k+1}(\rho)]
\]

\[
- \frac{1}{2} [u^k(\rho) + u^{k+1}(\rho)] + ag(u^k(\rho); R),
\]

where

\[
P_h \eta^k(\rho) = -\eta^k_0(\rho) - \frac{1}{\rho} \eta^k_0(\rho),
\]

\[
g(u; R) = \frac{u^2}{Q} + \frac{u^2}{R(1 + u^2)}.
\]

The difference scheme for solving (1.2) and (1.3) is

\[
\begin{align*}
D_h(s, 1) u^k(\rho) &= 0, \quad \rho \in \Omega_h, \quad k > 0, \\
u^k_+(0) &= u^k(1) = 0, \quad k > 0, \\
u^k(\rho) &= U_0(\rho), \quad \rho \in \Omega_h
\end{align*}
\]

(2.1)