DISPERSION OF LOVE WAVES IN A SPHERICAL EARTH WITH CORRUGATED SURFACE

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Summary — Propagation of Love waves over the spherical surface of a layered earth model has been discussed with special emphasis on the dispersion produced in the layer. The velocity of the waves with large wave-length increases appreciably as compared to the case of plane layer. The analysis has been extended to deduce an expression for the dispersion equation of the waves when the upper layer is of varying thickness. The modifications imparted to the dispersion equation depends on the amplitude only and not the shape of the corrugations provided we neglect small quantities of the second order. The effect is a substantial decrease in the phase velocity and becomes more pronounced if the amplitude of the corrugations increases.

Introduction. — Usually the sphericity of the earth is neglected in studying the surface waves produced by an earthquake shock. Since the curvature of the earth is small and the thickness of the crust is small compared to the radius of the earth the results obtained are fairly accurate, specially for waves of moderate wave lengths. But if the wave length of the surface waves is large the curvature of the earth's surface should be taken into account in the investigations and an examination of its effect on the nature of the waves is necessary. Jeans (1), regarding the earth as a gravitating compressible sphere arranged in concentric spherical layers has shown that all possible free vibrations fall into two distinct classes corresponding to the primary and secondary waves in seismology. In the particular case of the surface waves of the Rayleigh type he has been able to explain certain observed phenomena with his model. Sezawa (2) has obtained expressions for the displacements in Love-wave motion originated by a point source in a spherical layer surrounding a spherical medium. N. Jobert (2) has also investigated the same problem. They have concluded that the velocity of propagation of Love waves on a spherical surface is approximately equal to that on a plane surface even though the waves are relatively long, and that owing to the curvature of the sphere long period waves are dispersed to a greater extent. More recently, Scholte (6) has carried out investigations regarding different types of waves generated by a point source in a spherical earth model with the help of Saddle point method. But as he has pointed out that this otherwise helpful saddle point method fails to deal with the surface waves due to their large wave lengths.

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We have used the classical method adopted by SEZAWA and others to obtain expressions for the displacements as well as the dispersion equation in Love wave motion on a spherical earth surrounded by a spherical surface layer. For the more probable model in which the upper surface of the earth has corrugations of relatively small amplitudes we have extended the above results so as to obtain the modified dispersion equation. The corrugations have been assumed to be symmetrical about the origin of the earthquake waves. The primary disturbances have been supposed to be confined to a space the dimensions of which are negligible in comparison with all other occurring linear dimensions.

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Fig. 1

1. — Let $a$ be the radius of the free spherical surface of the earth and the thickness of the surface layer is $a-b$. We shall take the origin of the disturbance to be at the portion of the axis $\theta = 0$, of the spherical co-ordinates, just below the free surface and suppose that the waves are transmitted symmetrically in the direction of the co-latitude. Let $u, v, w$ be the radial, colatitudinal and azimuthal components of the displacements. Neglecting the effect of gravity the equations of motion are,

$$
\rho \frac{\partial^2 u}{\partial t^2} = (\lambda + 2\mu) \frac{\partial \Delta}{\partial r} - \frac{2\mu}{r \sin \theta} \frac{\partial}{\partial \theta} (w \sin \theta) + \frac{2\mu}{r \sin \theta} \frac{\partial w}{\partial \varphi},
$$

$$
\rho \frac{\partial^2 v}{\partial t^2} = (\lambda + 2\mu) \frac{\partial \Delta}{\partial r} - \frac{2\mu}{r \sin \theta} \frac{\partial v}{\partial \theta} + \frac{2\mu}{r} \frac{\partial w}{\partial r},
$$

$$
\rho \frac{\partial^2 w}{\partial t^2} = (\lambda + 2\mu) \frac{1}{r \sin \theta} \frac{\partial \Delta}{\partial \varphi} - \frac{2\mu}{r} \frac{\partial w}{\partial r} + \frac{2\mu}{r} \frac{\partial w}{\partial \theta},
$$

where $\rho$, $\lambda$, $\mu$ are the density and the Lamé's constants of the medium and

$$
\Delta = \frac{1}{r^2 \sin \theta} \left\{ \frac{\partial}{\partial r} (r^2 u \sin \theta) + \frac{\partial}{\partial \theta} (r v \sin \theta) + \frac{\partial}{\partial \varphi} (r w) \right\}.