THE DYNAMIC COMPUTATION OF CLOSED CYLINDRICAL SHELL UNDER IMPACT LOAD

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Abstract

This article discusses the dynamic computation of the closed cylindrical shell under impact load. In the text we analyse the changes of the momenta and the energy on each stage in the impact process, take into account the effect of the mass of impact object and the system of the closed cylindrical shell by impact, and transform the distributed mass of the whole cylindrical shell into an only concentrated "equivalent mass" by the method of reduced mass. Consequently we derive the dynamic factor of the closed cylindrical shell due to impact load.

The method proposed in this paper is of practical worth and is more convenient in calculations.

I. Introduction

The dynamic computation of the rods under impact load were discussed in references [1,2,3]. In this paper we will study the problems of the dynamic computation for the closed cylindrical shell under impact load by the principle of energy. In the discussion we take into account the effect of the mass of impact object and the closed cylindrical shell by impact, transform the shell, which possesses the distributed mass, into an elastic system having merely a concentrated mass by using the way of reduced mass, and accordingly we derive the dynamic factor of the closed cylindrical shell due to impact load.

For the purpose of simplifying the problems we assume as follows: neglecting the damping effects in impact system, without taking the loss of energy into account in the impact process, and not considering the local plastic deformation of bodies, on which by impact, the maximum dynamic deflections in shell are still in the linear range and the maximum dynamic stress doesn't exceed the proportional limit of materials of the shell.

II. Analyses of Impact Process

Let a closed cylindrical shell be set horizontally, as shown in Fig. 1. The length of the shell is L, the radius is R, thickness is h. Now let a weight Q strike towards a stationary shell, and the struck point be \( m_1 \). We choose the right-
handed orthogonal curvilinear coordinate system: $\alpha$, $\beta$, $\gamma$; they are along the directions of the generatrix, the circular arc and the radius respectively, in which the direction of $\gamma$ is positive when it moves towards the center of the circle. The coordinates of impact point $m_t$ is $(\alpha_t, \beta_t)$. Let us suppose that after impact the striking weight $Q$ attaches to the shell and becomes an integration, a dynamic deflection $w_{d_t}$ is produced at the point of impact in the shell and meanwhile the velocity of the weight $Q$ decreases to zero rapidly. If the mass of the impact object and the shell are $M$ and the $M$, respectively, let $V_0$ be velocity of weight $Q$ at the instance before impact. At the present moment the momentums of impact body is $MV_0$, then. If the shell obtains the velocity $V$ at the point of impact at the instance after impact, the momenta, which are gained by the shell at the instance after impact, do not equal $MV$, but are a portion of the quantity $MV$, since the velocities at other points on the shell are different from $V$, and what is more at the supports of the shell, these velocities will be equal to zero. Let us suppose that a part of the total mass $M$, of the shell $eM$, would be concentrated on the point of impact, where $e<1$, then the momenta the shell obtains are truly $eMV$, in which $e$ is called the reduced coefficient, which may be found by the equal principle of the kinetic energy in vibration, which is completed by the whole shell possessing the distributed mass primarily, and by an elastic system having a concentrated "equivalent mass" got by reduction.

Now we apply the theorem of conservation of the momentum at instance before impact and after impact for the struck system. The momenta of struck object at instance before impact possessing the quantity $MV_0$ must be equal to the momenta $(M+eM)V$, which are obtained by an integration composed of the struck object connected with the shell at the moment after impact. Then we find the velocity which is obtained by the shell at the impact point at instance after impact

$$V=MV_0/(M+eM)$$  \hspace{1cm} (2.1)

Moreover, we use the theorem of conservation of the energy on that stage from the instance after impact to the vanishing of velocity at impact point on shell. When the shell after impact has already got the velocity $V$ at the point of impact, then if the weight arrives at the extremity the velocity from $V$ will decreases to zero. If the set impact is due to the motion of the weight along the direction of radius of the shell, then its course is just the normal dynamic deflection $w_{d_t}$ at the point of impact on shell. If we neglect the potential energy which is produced by the displacements of $u$ and $v$ along the directions of the generatrix and the circular respectively, then the sum of kinetic energy and potential energy of both the striking object and the shell at instance after impact must be equal to the potential energy of the normal elastic resistance of thin shell while the striking point reaches the extremity, namely

$$\frac{1}{2}(M+eM)V^2 + Mgw_{d_t} = \frac{1}{2}Pw_{d_t}$$  \hspace{1cm} (2.2)

in which $P$ is the maximum normal elastic resistance in shell, $w_{d_t}$ is the maximum normal dynamic deflection at the impact point on shell. Since $w_{d_t}$ and $P$ increase from zero to maximum, on the right-hand side in equation (2.2) is the work done by the elastic resistance $P$, the so-called potential energy of elastic resistance.