NOISE AT INCEPTION AND COLLAPSE OF A CAVITY

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Abstract

The paper analyzes the noise at inception and collapse of an isolated bubble cavity filled with gas and vapour. The expressions and their numerical solutions of the sound pressure and the vibration velocity are presented.

The results indicate that the noise occurs at every stage of a cavity. The noise has comparatively big value only at the late period of collapse. The sound pressure is of magnitude 100db.

Employing the basic acoustic assumption[1], the noise at inception and collapse of an isolated bubble cavity filled with gas and vapour in an inviscid incompressible liquid with surface tension is analysed.

I. Basic Equation

The generalized Lighthill equation can be expressed as[2]

\[ \nabla^2 p - \frac{1}{c_0^2} \ddot{p} = -\dot{q} + \nabla \cdot \vec{f} - \frac{\partial^2 \tau_{ij}}{\partial x_i \partial x_j} \]  \hspace{1cm} (1.1)

where \( P \)—sound pressure;  
\( c_0 \)—sound velocity in liquid;  
\( q \)—fluctuating rate of mass in unit volume of liquid (here, the \( q \) results from the volume fluctuation of the bubble);  
\( \vec{f} \)—external fluctuating force done on unit volume of liquid;  
\( \tau_{ij} \)—stress tensor of liquid.

Here, the noise source only has relation to the mass fluctuation of the liquid. Formula (1.1) can be simplified

\[ \nabla^2 p - \frac{1}{c_0^2} \ddot{p} = -\dot{q} \] \hspace{1cm} (1.2)

As the size of the bubble is much less than the wavelength of the noise, the solution of formula (1.2) is

\[ p(r,t) = \frac{Q(t)}{4\pi r} \] \hspace{1cm} (1.3)

and
\[ Q = \int q dV = \frac{d}{dt}(\rho V) \]

where \( \rho \) — density of liquid;
\( t \) — time;
\( r \) — radial distance from bubble centre at any point of liquid;
\( V \) — volume of bubble at any moment.

Since the density fluctuation can be neglected, hence \( \rho = \rho_0 \) where \( \rho_0 \) is the density of the liquid at initial moment. Formula (1.3) can be simplified as

\[ p(r,t) = \frac{\rho_0 \dot{V}(t)}{4\pi r} \quad (1.4) \]

II. Determination of Sound Pressure

Suppose the radius of the bubble at any moment is \( R \). Hence

\[ V = \frac{4}{3}\pi R^3 \]

and

\[ \dot{V} = 4\pi (R^2 \dot{R} + 2R \dot{R}^2) \quad (2.1) \]

By reference [3] and [4], we have

\[
\dot{R} = \frac{2}{3\rho_0} (p_\infty - p_\nu) \left[ \left( \frac{R_0}{R} \right)^3 - 1 \right] - \frac{2p_1}{3(1-\gamma)\rho_0} \left[ \left( \frac{R_0}{R} \right)^3 - 1 \right]^{3/2} + \frac{2\sigma}{\rho_0 R} \left[ \left( \frac{R_0}{R} \right)^2 - 1 \right]^{1/2} \quad (2.2)
\]

where \( p_\infty \) — pressure in liquid at infinity; \( p_\nu = p_\nu(T) \) — vapour pressure in bubble; \( \sigma = \sigma(T) \) — surface tension of liquid; \( \gamma \) — gas constant of air; \( p_1 \) — air pressure in bubble at initial moment; \( R_0 \) — initial radius of bubble; \( T \) — temperature of liquid.

Formula (2.2) is satisfied for inception and collapse of the bubble. However, the quantities of \( R_0, p_\infty \) and \( p_1 \) are different for different stages.

Substituting (2.1) and (2.2) into (1.4), we obtain

\[
p(R,r) = \frac{\sigma}{r} \left[ \left( \frac{R_0}{R} \right)^2 - 3 \right] + \frac{(p_\infty - p_\nu)}{r} \left[ \frac{4R}{3} \left[ \left( \frac{R_0}{R} \right)^3 - 1 \right] - \frac{R_0^3}{R^2} \right] \]

\[
- \frac{p_1}{(1-\gamma)R} \left[ \frac{4R}{3} \left[ \left( \frac{R_0}{R} \right)^3 - \left( \frac{R_0}{R} \right)^{3\gamma} \right] - \frac{R_0^3}{R^2} + \frac{\gamma R_0^{3\gamma}}{R^{3\gamma-1}} \right] \quad (2.3)
\]