AN OPTIMAL CONTROL MODEL
OF THE LIFE-CYCLE
RESEARCH PRODUCTIVITY OF SCIENTISTS

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(Received May 30, 1986)

A continuous time model using optimal control techniques is presented which implies that a scientist's productivity will eventually decline with age. This implication is at variance with Cole's empirical findings but is consistent with Diamond's empirical findings.

In recent work a discrete time model due to Becker was specialized to explain the research productivity of scientists. Here mathematically more sophisticated optimal control techniques are used to more rigorously derive similar implications in a continuous time framework. The model presented here is essentially an adaptation of a simplified version of a model first presented by Ben-Porath.

At any age $i$, the model has the scientist maximizing $U$, the present value of his disposable income:

$$U = \int_{i}^{n} e^{-rt}[WK(t) - WSK(t)]dt$$

where $K$ is his stock of prestige capital, $W$ is the rate of return to prestige capital, $i$ is the scientist's current age, $n$ is the scientist's age at retirement, and $S_t$ is the proportion of time in period $t$ that is devoted to the production of prestige capital. $S_t$ is constrained to be in the closed interval $[0,1]$. In order to satisfy the Weierstrass condition for a maximum we assume that the integrand is a concave function of $S_t$.

Let the production function for prestige capital be:

$$Q_t = \beta (S_t K_t)^a$$

where $Q_t$ is the flow of prestige capital. For some purposes the inclusion of non-time inputs is important, but we ignore them here.
The rate of change of the capital stock is given by:

\[ \dot{K}_t = Q_t - \delta K_t \]  

(3)

where a dot above the \( K \) indicates a derivative with respect to time and where \( \delta \) is the rate by which the stock of prestige capital deteriorates.

The Hamiltonian for this problem is:

\[ H = e^{-rt}(1 - S) W K + \lambda (Q - \delta K) \]  

(4)

where the costate variable \( \lambda \) is the discounted shadow price of investment in prestige capital. The first of the relevant necessary conditions for a maximum is:

\[ \dot{\lambda} = -\frac{\partial H}{\partial K} = -e^{-rt}(1 - S) W \lambda \left( \frac{Q}{K} - \delta \right) \]  

(5)

In terms of current prices \( \mu = \lambda e^{-rt} \). So the condition can be re-written as:

\[ \dot{\mu} = -(1 - S) W - \mu \left( \frac{Q}{K} - \delta - r \right) \]  

(6)

The second of the relevant necessary conditions for a maximum is:

\[ 0 = \frac{\partial H}{\partial S} = -e^{-rt} W K + \mu e^{-rt} \frac{Q}{S} \]  

(7)

The transversality condition is that:

\[ \lambda (n) = 0 \]  

(8)

Substituting Eq. (2) into Eq. (7) we can eventually obtain:

\[ Q = (\beta) \frac{1}{1 - \alpha} \left( \frac{\mu a}{W} \right)^{\frac{\alpha}{1 - \alpha}} \]  

(9)

Substituting Eq. (7) into Eq. (6) we obtain the simple differential equation:

\[ \dot{\mu} = -W + \mu (\delta + r) \]  

(10)