NONLINEAR VIBRATION OF CIRCULAR CORRUGATED PLATES*

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Abstract

In this paper, first by using Hamilton principle, we derive the variational equation of circular corrugated plates. Taking the central maximum amplitude of circular corrugated plates as the perturbation parameter and adopting the perturbation variational method, in the first-order approximation, we obtain the natural frequency of linear vibration of circular corrugated plates and then the nonlinear natural frequency of the corrugated plates. By comparing with the linear results, the attempt of this paper is proved feasible.

I. Introduction

Circular corrugated plates play an important part in elastic sensor units of precise instruments, but there are only a few scholars who study the dynamic problems of these elastic units. The reason that causes this situation is: 1) the large deflection equations of plates are nonlinear, and up to now the approximate solution under the condition of static equilibrium is still not easily found\(^2,3,4\); 2) the shapes of corrugated plates are complicated, and there are more parameters concerned\(^5,6\); moreover the term inertial force makes the problem more difficult. This paper which is based on papers \(4,7\) is to carry on further study on the dynamic problem. As we know, the stiffness of circular corrugated plates along the radial differs largely from the stiffness along the tangential direction, as far as the structure is concerned, this is analogous to anisotropic plates. In this paper, the anisotropic plate is used instead of the corrugated plate\(^6\), and selecting different parameters will arrive at one of the dynamic problems of the sine corrugation, the sawtooth corrugation and the trapezoid corrugation types, and so on. Using the perturbation variational method\(^8,9,10,11,12\), we derive the approximate expression of the nonlinear natural frequency of circular corrugated plates, and furthermore from this result, the nonlinear natural frequency expression of circular flat plates can be derived.

II. Combined Problem of Circular Corrugated Thin Plates

In this problem, there are three kinds of potential energies\(^11\):

the potential energy of bending

\[
V_1 = \int_0^\alpha \frac{1}{2} (M_rK_r + M_\theta K_\theta) 2\pi r dr
\]  

(2.1)

the strain energy of membrane

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the potential energy lost by the loads

\[ V_s = -\int_0^a qw 2\pi rdr \]  \hspace{1cm} (2.3)

and the kinetic energy of the plates is

\[ T = \frac{1}{2} \int_0^a \rho \left( \frac{\partial w}{\partial t} \right)^2 2\pi rdr \]  \hspace{1cm} (2.4)

where \( a \) is the radius of the circular corrugated plate, \( \rho \) is the mass per area of the plate.

From papers [5,6], we have

\[ M_r = -Dk_r^{-1} \left( \frac{\partial^4 w}{\partial r^4} + \mu \frac{\partial^2 w}{\partial r^2} \right) \]  \hspace{1cm} (2.5a)
\[ M_\theta = -Dk_r^{-1} \left( \lambda^2 \frac{1}{r} \frac{\partial w}{\partial r} + \mu \frac{\partial^2 w}{\partial r^2} \right) \]  \hspace{1cm} (2.5b)
\[ \varepsilon_r = \frac{\partial u}{\partial r} + \frac{1}{2} \left( \frac{\partial w}{\partial r} \right)^2 = \left( k_r^2 N_r - \mu N_\theta \right) / Ehk_z \]  \hspace{1cm} (2.6a)
\[ \varepsilon_\theta = u / r = (N_\theta - \mu N_r) / Ehk_z \]  \hspace{1cm} (2.6b)

and

\[ K_r = -\frac{\partial^3 w}{\partial r^3}, \hspace{1cm} K_\theta = -\frac{1}{r} \frac{\partial w}{\partial r} \]

in which \( \lambda^2 = k_1 k_2^\prime, \hspace{1cm} k^2 = k_1 k_2, \hspace{1cm} D = Eh^2 / 12(\lambda^2 - \mu^2), \hspace{1cm} k_1, k_2, k_1^\prime, k_2^\prime \) are the parameters concerned with the radial and the tangential directions of the circular corrugated plate.

According to Hamilton principle, the quantity

\[ \Pi = \int_{t_1}^{t_2} (T - V_1 - V_2 - V_3) dt \]  \hspace{1cm} (2.7)

should take the stationary value.

For the vibration of the circular corrugated thin plate, \( w(r,t) \) should be a periodic function, we take

\[ w(r, t) = y(r) \cos \omega t \]  \hspace{1cm} (2.8)

where \( \omega \) is the natural frequency of the periodic motion, \( \omega = 2\pi f, \ f \) is the frequency. Because the phase angle has no effect upon our discussion on the problem, we take it to be zero.

Let \( t_1 - t_2 \) be just a period \( 2\pi / \omega \) and \( \tau = \omega t \), then

\[ \Pi = \frac{1}{\omega} \int_0^{2\pi} (T - V_1 - V_2 - V_3) d\tau \]

\[ = \frac{\pi}{\omega} \int_0^a \left\{ \rho \omega^2 \left( \frac{\partial w}{\partial \tau} \right)^2 + 2qw + M_r \frac{\partial^2 w}{\partial \tau^2} + M_\theta \frac{1}{r} \frac{\partial w}{\partial r} \right\} r dr d\tau = \frac{\pi}{\omega} \Pi_1 \]  \hspace{1cm} (2.9)