FIXED PANSYSTEMS THEOREMS AND PANSYSTEMS
LOGIC CONSERVATION OF CHAOS

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Abstract

The study of nonlinear problems was developed in works\cite{1,2} by means of the pansystems methodology (PM) which does not need the condition related to differential manifolds and linear space. In work \cite{2}, within the framework of PM, we proved that study of panchaos, panattractor and strange panattractor can be transformed conditionally into some forms of fixed subsets. As a continuation of work \cite{2}, we now research the pansystems logic conservation of panchaos, panattractor, strange panattractor and some other fixed subsets.

Within the framework of PM, we have indicated how the studies on nonlinear problems can be transformed into one of fixed pansystems. It is known that the fixed pansystems theorems complement or develop traditional theory of fixed point, and do not need some stricter terms such as topolocity, linearity, continuity, compactness, convexity, etc.. In this paper, we consider the further problems of pansystems logic conservation of panchaos, panattractor, strange panattractor and some other fixed subsets.

I. Panchaos, Panattractor and Strange Panattractor

We first introduce some notations and results which will be required in the following studies. Let $G$ be nonempty set, $P(G)$ denote the set of all subsets of $G$, $g \in P(G)$ (or $g \subseteq G^2$) be called a binary relation on $G$, $I = \{(x, x) \mid x \in G\}$, $g^{-1} = \{(x, y) \mid (y, x) \in g\}$. If $x \in G$, $D \subseteq G$ we can define

$$
\begin{align*}
\langle x \circ g \rangle &= \{y \mid (x, y) \in g\}, \\
D \circ g &= \bigcup_{x \in D} x \circ g, \\
g \circ x &= \{y \mid (y, x) \in g\}, \\
g \circ D &= \bigcap_{y \in D} g \circ y = \{x \mid \forall y \in D, (x, y) \in g\}
\end{align*}
$$

Similarly, we can also define $g \circ y$, $g \circ D$, $x \circ g$ and $D \circ g$.

**Definition** If $g \subseteq G^2$, $D \subseteq G$, $D^2 \cap g = \emptyset$, then $D$ is called panchaos for $g$. If for all $x \in G - D$, $x \circ g \cap D = \emptyset$, then $D$ is called panattractor for $g$. If $D$ is both panchaos and panattractor for $g$ then $D$ is called strange panattractor for $g$.

The following Theorems 1 to 5, which have been proved in work\cite{2}, are necessary for the further studies. Let $g \subseteq G^2$, $D \subseteq G$.

**Theorem 1** The subset $D$ of $G$ is a panchaos for $g$, if and only if for any $x, y \in D$ the
relations \((x,y) \in g\) and \((y,x) \in g\) are valid.

**Theorem 2** The subset \(D\) of \(G\) is a panattractor for \(g\) if and only if for \(x \in \overline{D}\), there exists a \(y \in D\) such that \((x,y) \in g\).

**Theorem 3** In order that the subset \(D\) of \(G\) is a panattractor for \(g\) it is necessary and sufficient that \(D \subseteq g \ast D\).

**Theorem 4** In order that the subset \(D\) of \(G\) is a panattractor for \(g\) it is necessary and sufficient that \(D \supseteq g \ast D\).

**Theorem 5** In order that the subset \(D\) of \(G\) is a strange panattractor for \(g\) it is necessary and sufficient that \(D = g \ast D\).

Theorems 3 to 5 show that the panchaos, panattractor, strange panattractor for \(g\) correspond to three classes of fixed subset respectively:

\[
\begin{align*}
FS_1(g) &= \{D \mid D \subseteq G, D \neq \emptyset, D = g \ast D\} \\
FS_2(g) &= \{D \mid D \subseteq G, D \neq \emptyset, D \supseteq g \ast D\} \\
FS_3(g) &= \{D \mid D \subseteq G, D \neq \emptyset, D \supseteq g \ast D\}
\end{align*}
\]

Some existence conditions of fixed subset are given by the following theorems.

**Theorem 6** If \(g \ast G \neq \emptyset\), and if for \(x \in g \ast G\) there exists \(y \in g \ast x\) such that \((x,y) \in g\), then \(g \ast G \in FS_1(g)\).

**Theorem 7** Let the hypothesis of Theorem 6 be fulfilled and let the relation \(g\) be transitive, then there exists one and only one subset \(D\) of \(G\) such that \(D = g \ast D\), and \(D = g \ast G\).

**Theorem 8** If there is \(D \subseteq G\) such that \(\bigcap_{i=1}^{\infty} g^{(n-1)} \ast D = \emptyset\), then \(FS_1(g) \neq \emptyset\).

Here

\[
g^{(n)} \ast D = g \ast (g^{(n-1)} \ast D), \quad g^{(2)} \ast D = g \ast g \ast D, \quad g^{(0)} \ast D = D.
\]

**Theorem 9** If there is \(D \subseteq G\) such that \(\bigcup_{n=1}^{\infty} g^{(n)} \ast D = \emptyset\), then \(FS_1(g) \neq \emptyset\).

**Theorem 10** Let \(D \subseteq G\), \(D \neq \emptyset\), and let \(c(x)\) be characteristic function of \(D\), then

1) \(c(x) \geq \min\{c(y) \mid y \in g \ast x\}\) if and only if \(D \in FS_1(g)\).

2) \(c(x) \leq \min\{c(y) \mid y \in g \ast x\}\) if and only if \(D \in FS_1(g)\).

3) \(c(x) = \min\{c(y) \mid y \in g \ast x\}\) if and only if \(D \in FS_1(g)\).

**Theorem 11** If \(D\) is a panchaos for \(g\), then \(g \ast D\) is a panattractor for \(g\). Conversely, if \(D\) is a panattractor for \(g\), then \(g \ast D\) is a panchaos for \(g\).

**Theorem 12** Suppose that \(f, g \subseteq G^2\), \(D_1 \in FS_1(f)\), \(D_2 \in FS_1(g)\), \(D_1 \cap D_2 \neq \emptyset\), then \(D_1 \cup D_2 \in FS_1(f \cup g)\), \(D_1 \cap D_2 \in FS_1(f \cup g)\).

For

\[
g \ast D = \{x \mid y \in D, (x,y) \in g\}
\]

we have

**Theorem 13** If for all \(x \in G\), \(g \ast x \neq \emptyset\), then there is \(D \subseteq G\), such that \(D = g \ast D\).

**Theorem 14** If there is \(D_1 \subseteq G\), \(D_1 \subseteq g \ast D_1\), then there exists \(D \subseteq G\), such that \(D = g \ast D\).

II. Logic Conservation

In this section we study the pansystems logic conservation of the panchaos, panattractor,