ANALYSIS OF THE MOTION OF A GYRO-THEODOLITE

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Abstract

With the method of analytical mechanics, this paper studies the motions of a gyro-theodolite under the action of (1) the torque of gravity only, (2) the torque applied by the band suspension, (3) the torque of the band suspension with air damping considered, the equations of motion are then established and their solutions are found. Furthermore, analysis of the law of motion and the behaviour of gyro-theodolite during the orientation is made.

Gyro-theodolite is a main instrument for orientation in mine surveying. An analysis about the dynamics of orientation in Refs. [5], [6], [10] are not quite rigorous, or even involve mistakes. With the method of analytical mechanics, this paper studies the motions of gyro-theodolite under the action of (1) the torque of gravity only, (2) the torque applied by the band suspension, (3) the torque of the band suspension with air damping considered, the equations of motion are then established and their solutions are found. Furthermore, analysis of law of motion and the behaviour of gyro-theodolite during the orientation is made. The results analysed in this paper may be useful to correct the errors in some literature, to improve the computational method about the orientation of a gyro-theodolite and to make the computation more accurate. Moreover, they may be valuable for analysing the structure and the quality of a gyro-theodolite.

I. A Mechanical Model and Kinematical Analysis

A gyro-theodolite is composed of two parts, a gyroscope used as the sensitive element and a theodolite which orientated by the sensitive element. It can be simplified as a mechanical model as follows: a rotor of the gyroscope is installed in a rotor casing, on which the rotor axis and a suspending pole is fixed. The casing is freely suspended under the box of the instrument with a flexible metal band through the pole. We denote the distance of the centre of gravity of the casing from the suspension point O by a, and assume this instrument to be fixed to the surface of the earth at latitude \( \varphi \) for operating.

We choose three coordinate systems described below (Fig. 1, Fig. 2):

1. a translatory coordinate system \( O\xi\eta\zeta \), with the suspension point \( O \) as the origin and translating in an inertial reference frame;

2. a directing coordinate system \( O\xi\eta\zeta_{0} \), fixed on the earth, with point \( O \) as the origin, and rotating with angular velocity \( \omega \) of the earth's rotation in the translatory coordinate system;

3. a Resal's coordinate system \( Oxyz \), fixed on the casing, with point \( O \) as the origin, its axis \( Oy \) having the same direction as the angular momentum \( \mathbf{H} \) of the rotor rotation. With this system, it
is possible to separate the rotation of the rotor about its axis, so that we can devote our attention to study the motion of the rotor axis in the directing coordinate system \( O_{\xi_0\eta_0\zeta_0} \) fixed on the earth. This motion can be indicated by the rule of the change of the Resal’s angles \( \alpha, \beta \), with which the Resal’s system turns about the directing coordinate system. We consider that the true north \( \eta_0 \) in directing system is the direction which the rotor axis should be orientated.

The position of the Resal’s system \( O_{\xi_0\eta_0\zeta_0} \) is obtained by turning the directing coordinate system \( O_{\xi_0\eta_0\zeta_0} \) in such a way:

\[
(O_{\xi_0\eta_0\zeta_0}) \frac{\partial \zeta_0}{\partial t} \rightarrow (O_{\eta_0})
\]

\[
(O_{\eta_0}) \frac{\partial \eta_0}{\partial t} \rightarrow (O_{\xi_0\eta_0\zeta_0})
\]

The transformation matrices are

\[
\pi_\alpha' = \begin{pmatrix}
\cos \alpha & -\sin \alpha & 0 \\
\sin \alpha & \cos \alpha & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

\[
\pi_\beta' = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \beta & -\sin \beta \\
0 & \sin \beta & \cos \beta
\end{pmatrix}
\]

\[
\pi_\alpha' \cdot \pi_\beta' = \begin{pmatrix}
\cos \alpha & -\sin \alpha \cos \beta & \sin \alpha \sin \beta \\
\sin \alpha \cos \beta & -\sin \beta & -\cos \alpha \sin \beta \\
0 & \sin \beta & \cos \beta
\end{pmatrix}
\]

The angular velocity with which the Resal’s system rotates in the directing system is:

\[
\omega = \dot{\alpha} \zeta_0 + \beta \eta_0
\]

\[
= \beta \eta_0 + \dot{\alpha} \sin \beta \eta_0 + \dot{\alpha} \cos \beta \zeta_0
\]

(1.1)