A HIGH ORDER METHOD FOR NON-SMOOTH FREDHOLM EQUATIONS

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Abstract

In this paper, we deal with a class of the second kind of non-smooth Fredholm integral equations, which are related closely to Wiener-Hopf equations. Using Sloan's iterative technique, we obtain the superconvergent approximations. By means of the correction and collocation methods, we present a kind of iterative correction collocation approximations for this kind of equations, and show that this method is not only a high order and more simple method but also an adaptable one (see e.g. [11]).

Key words. Collocation approximation, correction, adaptable method

1. Introduction

Consider the equation

\[ u(s) - \int_0^1 k(s,t)u(t)\, dt = f(s), \quad s \in [0,1], \]

where \( k(s,t) = k(\tau) \mathbb{1} \) and \( f(s) \) are given, \( u \) is the unknown solution. Since it is related closely to Wiener-Hopf equations and is very important in practice, there are many numerical results about it (e.g. [1-11]). It is well known that the accuracy of the approximations of the second kind of Fredholm integral equations may be raised by Sloan iterations. Recently, Shi [9] presented a kind of iterative correction method for smooth Fredholm integral equations to raise the order of the Sloan's iterative approximation, for example, from \( O(h^r) \) to \( O(h^{2r}) \).

In this paper, similarly to [3], using Sloan iterative technique, we shall show that the Sloan iterative approximation is superconvergent for this equation with some singularity. By means of a kind of correction method, we shall show that one step of correction can raise the accuracy of the Sloan iterative approximations, for example, from \( O(h^r) \) to \( O(h^{2r}) \) with a little more additional computation, and this adaptable process can be continued for

Received February 18, 1993. Revised May 21, 1995.
any steps (see, e.g. [11]). This method can be applied to many other problems; one such application in elastostatics is given in Section 3.

2. The Main Results

For $\beta \in [0, 1)$, define the integral operator $K_\beta$ by

$$K_\beta u(s) = \int_\beta^1 k(s, t)u(t) \, dt,$$

and let $K_0 = K$. Then (1) can be written in operator form $(I - K)u = f$. We shall assume there exists a constant $\alpha^* \in (0, 1)$ such that

$$\int_0^{+\infty} t^{k-\alpha} |D^k k(t)| \frac{dt}{t} < \left\{ \begin{array}{ll} 1, & k = 0, \alpha \in (-\alpha^*, +\alpha^*), \\ +\infty, & k \geq 0, \alpha \in [0, \alpha^*). \end{array} \right. \quad (A)$$

For $k \geq 0, \alpha \in [0, 1)$, define norms

$$\|v\|_{C^k_\alpha} = \max_{0 \leq s \leq 1} \sup_{0 < l < k} \{|D^l u(s)| : s \in [0, 1]\},$$

and define the Banach space $C^k_\alpha$ to be the completion of smooth functions under the norm $\| \cdot \|_{C^k_\alpha}$. When $k = \alpha = 0$, $C^0_\alpha$ is the usual continuous functions space with uniform norm $\|u\|_{C^0_\alpha} = \|u\|_{\infty} = \sup |u|$. $C$ will denote a constant in this paper, it is always independent of the number of the mesh points. We employ a graded mesh:

$$0 = t_0 < t_1 < \cdots < t_n = 1,$$

where $e_i = [t_i, t_{i+1}]$, $h_i = t_{i+1} - t_i$, $h = \frac{1}{n}$, $t_{i+1} \leq C t_i$, $h_i \leq C h^{1-1/q}$, $h_i/t_i \leq C/(i - 1)$; for any fixed $i_0 \in \{0, 1, \ldots, n\}$, $t_{i_0} \leq C h^q$, where the refinement parameter $q, q > 0$, will be determined below.

For example, choose $t_i = (i/n)^q$; then $\{t_i\}$ satisfy (2).

For a given collocation points set $\{s_j : j = 0, 1, \ldots, m\}$ on $[0, 1]$, let $t_{ij} = t_i + s_j h_i$, $S_m = \{v : v_i, the restriction of v on e_i is a polynomial of order m, e_i \leq i; v_i = 0, i < i_0\}$, then we have an interpolatory projection $P_h : C^0_\alpha \rightarrow S_m$ defined by

$$P_h u(t_{ij}) = u(t_{ij}), \quad (i, j) \in Q = \{(i, j) : i_0 \leq i, j = 0, 1, \ldots, m\},$$

$$P_h u_i = 0, \quad i < i_0,$$

where the constant integer $i_0 \in \{0, 1, \ldots, n\}$ will be determined below. It is easy to see that $\|P_h\|_{\infty}$ is uniformly bounded. We also have a quadrature rule:

$$\int_0^1 v(t) \, dt \approx \int_0^1 P_h v(t) \, dt.$$

The order $r$ is defined by: For all $e_i$,

$$\left| \int_{t_i}^{t_{i+1}} v(t)(I - P_h)u(t) \, dt \right| \leq C \sum_{i=0}^{r-1} h_i^r \int_{t_i}^{t_{i+1}} ||D^{r-l}u||_\infty ||D^l v||_\infty \, dt.$$