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# Metastrategy simulated annealing and tabu search algorithms for the vehicle routing problem

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## Abstract

The vehicle routing problem (VRP) under capacity and distance restrictions involves the design of a set of minimum cost delivery routes, originating and terminating at a central depot, which services a set of customers. Each customer must be supplied exactly once by one vehicle route. The total demand of any vehicle must not exceed the vehicle capacity. The total length of any route must not exceed a pre-specified bound. Approximate methods based on descent, hybrid simulated annealing/tabu search, and tabu search algorithms are developed and different search strategies are investigated. A special data structure for the tabu search algorithm is implemented which has reduced notably the computational time by more than 50%. An estimate for the tabu list size is statistically derived. Computational results are reported on a sample of seventeen bench-mark test problems from the literature and nine randomly generated problems. The new methods improve significantly both the number of vehicles used and the total distances travelled on all results reported in the literature.

**Keywords:** Local search, approximate algorithms, heuristics, hybrid algorithms, simulated annealing, tabu search, vehicle routing problem.

## 1. Introduction

The vehicle routing problem (VRP) under capacity and distance restrictions involves the design of minimum cost delivery routes for a fleet of vehicles, originating and terminating at a central depot, which serves a set of customers. Each customer is supplied by exactly one vehicle route. The total demand of any vehicle route must not exceed the vehicle capacity. The total length of any route includes the inter-customer travel times and service times must not exceed a prespecified bound. Figure 1 provides an illlustration of this type of problem.

The following notations are used for representing the problem:

$n$ = the number of customers;

$N$ = the set of customers, $N = \{1, \dots, n\}$;

$q_i$ = the demand of customer $i \in N$ ($i = 0$ denotes the depot, $q_0 = 0$);

$\delta_i$ = the service time of customer $i \in N$ ($\delta_0 = 0$);

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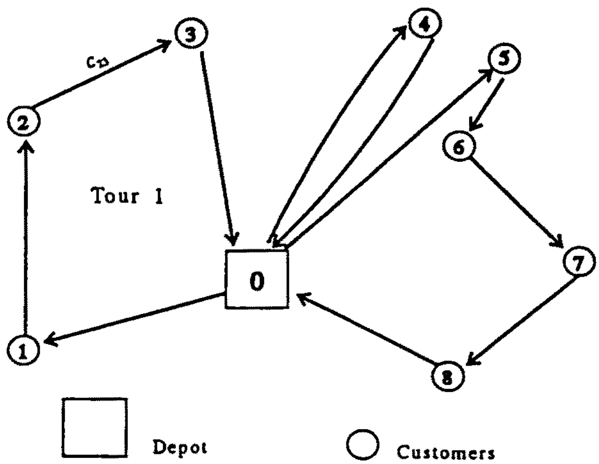


Fig. 1. The vehicle routing problem.

$c_{ij}$ = the travel time (distance) between customers $i$ and $j$, $c_{ij} = c_{ji}\ \forall i,\ j \in N$ ($c_{ii} = \infty,\ \forall i \in N$);
$v$ = the number of vehicles, which is a *decision* variable in our problem;
$V$ = the set of vehicles, $V = \{1, \ldots, v\}$;
$Q$ = the vehicle capacity;
$R_p$ = the set of customers serviced by vehicle $p$;
$C(R_p)$ = the cost (length) of the optimal travelling salesman tour $\pi_p$ over the customers in $R_p \cup \{0\}$. This cost includes the travel times ($c_{ij}$) and the service times ($\delta_i$);
$L$ = the prespecified upper bound on the maximum tour length;
$S$ = the feasible solution which is defined as $S = \{R_1, \ldots, R_v\}$;
$C(S)$ = the total sum of each individual tour length $C(R_p)$ for all $p \in V$.

Our goal is to find an optimal solution (say $S$ without loss of generality) that minimizes the total travel length and satisfies:

$$\begin{aligned}
&\bigcup_{p=1}^{v} R_p = N, && R_p \cap R_q = \emptyset, \forall p \neq q \in V;\\
&C(R_p) = \sum_{i \in R_p \cup \{0\}} (c_{i\pi(i)} + \delta_i) \le L, && \forall p \in V;\\
&\sum_{i \in R_p} d_i \le Q, && \forall p \in V;\\
&C(S) = \sum_{p \in V} C(R_p),
\end{aligned} \tag{1}$$